

N65-29462

FACILITY FORM 802

(ACCESSION NUMBER)

174

(PAGES)

CR 58215

(NASA CR OR TMX OR AD NUMBER)

(THRU)

(CODE)

33

(CATEGORY)

SUMMARY TECHNICAL REPORT

NASA GRANT NG-428

MAY 1964

Cat. 30

UNPUBLISHED PRELIMINARY DATA

GPO PRICE \$ \_\_\_\_\_

CFSTI PRICE(S) \$ \_\_\_\_\_

Hard copy (HC) 3.00

Microfiche (MF) .75

ff 653 July 65

DEPARTMENT OF PHYSICS

GEORGETOWN UNIVERSITY

WASHINGTON, D.C.

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## Introduction

The work reported in this paper is concerned with the problem of controlling the temperature of a spacecraft surface by means that do not put undue demands on either the power requirements or the weight. The basic approach taken was to find materials whose ratio of absorptance of solar radiation and emittance could be changed in such a way that the total heat input into the surface can be balanced by appropriate re-emission of radiation. Ideally, one would want to find a system of materials whose ratio of absorptance ( $\alpha$ ) to emittance ( $\epsilon$ ) can be regulated in such a way that the temperature of the space craft is controllable. To this end a number of materials were examined and a semi-empirical theory that explains the heat transfer in layers of homogeneous materials was developed.

The samples reported on in the following are two types of electroluminescent materials - one Sylvania "Nite-Lite" and one Sierraglow panel - one Nickel-Barium photo-voltaic cell, one Copper-Lead Sulphide photo-conductive cell, and one n-type Germanium sample with a doping concentration of about  $10^{16}$  donors per cc; i.e., a non-degenerate sample. Finally, the possibility of using electro-optical materials was investigated by simulating their effects with polaroids.

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## Experimental Procedures

The experimental procedures that were followed in this work have been discussed in great detail elsewhere <sup>(1)</sup>. For completeness an abbreviated description is given in the following.

The samples to be examined were suspended in a space simulation chamber whose walls were kept near liquid nitrogen temperature. The temperatures at the wall and the samples themselves were monitored constantly by placing thermocouples appropriately. As the samples were irradiated with a solar simulator beam of intensity  $I$  the following heat rate equation was true.

$$m C(T) \frac{dT}{dt} = a_s(T)I - A_s(T)\sigma T^4 + P \quad \dots \quad (1)$$

Here  $m$  and  $T$  are the mass and temperature of the sample;  $C(T)$  is its specific heat;  $a$  its projected area; i.e., that area of the sample that intercepts the intensity  $I$  of the solar simulator;  $\alpha(T)$  and  $\epsilon(T)$  are the absorptance and emittance of the sample and  $A$  its total area.  $P$  finally is the radiation absorbed by the sample other than the solar simulator intensity; for example, the radiation from the chamber walls. In order to determine  $\alpha(t)$  the samples are irradiated with two solar simulator intensities  $I_1$  and  $I_2$ . By writing equation (1) twice for these two intensities and subtracting one from the other, one obtains

$$\alpha(T) = \frac{mC(T)}{a(I_1 - I_2)} \left[ \left( \frac{dT}{dt} \right)_1 - \left( \frac{dT}{dt} \right)_2 \right] \quad \dots \quad (2)$$

In this relation a term  $P_1 - P_2$  has been left out because it constitutes merely a minor correction to  $\alpha(T)$ . After the sample obtained thermal equilibrium the solar simulator was turned off and the sample was allowed to cool down to its original equilibrium temperature. Equation

(1) reduces then to

$$mc(T) \frac{dT}{dt} = P - A\epsilon(T)\sigma T^4$$

which can be written as (2)

$$\epsilon(T) = \frac{mc(T) \frac{dT}{dt}}{A\sigma(T^4 - T_v^4)} \quad (3)$$

where  $T_v$  is the temperature of the walls. In principle all values in equations (2) and (3) are known with the exception of  $\frac{dT}{dt}$  which was measured off a strip chart recording. The values of  $\alpha(T)$ ,  $\epsilon(T)$ , and  $\alpha/\epsilon$  were obtained by using a computer.

### Absorptance and Emittance of Electroluminescent Materials

Two commercially available samples were examined. One was a Sylvania "Nite-Lite" which consisted of an iron plate, 3" in diameter, to which an electroluminescent, a transparent conducting, and a protective fritted glass layer were attached. The emittance and absorptance and the spectrum of this sample were examined under a variety of conditions.

Referring to equation (1), it is clear that the term  $P$  should now also include the electrical power put into the sample, due to the application of an alternating voltage. It is much less clear in which form this power term should be written since a large part of it is converted into visible light. In order to be able to write  $P$  as a sum of one emissive and one absorptive term one has to know the initial and final state of the system. An additional difficulty arises from the fact that the light output and the interaction between the solar simulator input and the electrical input is a function of temperature. In figures (1) and (2) three spectral curves are represented for sample temperatures of 300°K and 100°K, respectively. The lower curves (run A) represent the spectral emission of the Sylvania sample with an applied 60 cycle voltage of 370 volts, peak to peak. The next higher curve (run B) represents the spectrum of the solar simulator as reflected from the sample, and the highest curve is the spectrum obtained when the solar simulator light was reflected from the sample as it was in state of electroluminescence (run C).

From the results it is clear that the intensities do not add up in a simple fashion. Instead the sum of the intensities of runs A and B is less than the intensity of run C for a sample temperature of 300°K and more than the intensity of run C for a sample temperature of 100°K.

Due to these complications, it was decided to neglect the electrical power input completely and interpret the measured values of  $\alpha$  and  $\epsilon$  as absorptances and emittances of the samples in the state of electroluminescence.

The results so evaluated are represented in Figs. 3 and 4. In case of the run without an applied voltage, the sample was first cooled down; then the solar simulator beam was turned on until the sample reached its new equilibrium temperature. At this stage the beam was turned off and the sample allowed to cool down. As discussed above, one obtains  $\alpha$ ,  $\epsilon$  and  $\alpha/\epsilon$  by repeating this run for two different intensities. In case of the 60 cps applied voltage, the sequence of experimental procedures was the same as just described except that the initial and final equilibrium temperature was not determined by the conditions in the chamber alone but also by the electrical power input. Figure 3 demonstrates clearly that the pertinent parameters do not change much in the available temperature range of 140°K to 220°K, while  $\epsilon$  changes by a factor of two with an applied voltage over the same temperature range.  $\alpha$  on the other hand is much less affected by the field. Although an analysis of these results cannot be given at the present time, it is clear that any model of electroluminescence should be able to explain why the absorption process hardly changes at all, while the emission process appears to be critically dependent on the externally applied field. These results finally give rise to a large change in the slope of the  $\alpha/\epsilon$  curve as a function of temperature.

Since there have been a large number of publications <sup>(3,4,5)</sup> on the light emission of single crystals of luminescent materials, several

of the reported experiments were repeated in order to determine whether the models that explain the single crystal results may be applied also to a crystalline powder. To this end, the Sylvania sample was excited by square wave pulses of 100 volts with a rise time of  $0.5\mu$  sec. The light generated in the sample was observed with RCA 6342 A and 1P-21 photomultiplier tubes. The applied pulses and the amplified signal from the photomultiplier were displayed on a Tektronix 581 oscilloscope. A typical pattern is reproduced in Fig. 5. The repetition rate of the pulses was varied from 100 cps to 12 keps. In addition, a variety of Corning color filters were used to attempt to determine relaxation times belonging to various types of transitions. The result of these observations is given in Table 1. The  $\tau$ 's are defined in Fig. 5 and given in Table 1. It was found that the square wave fields produce maximum luminescence whenever the sample fields were changing. The maximum light output follows the square wave rise by 30-70 $\mu$ sec. The light output then decreases while the applied field is steady because the effective internal fields were decreasing due to the polarization of the sample. Generally, the light output is made up of two broad spectra; one located in the green and the other in the blue part of the spectrum. Observations made with a yellow and a violet filter suggest that the transitions producing the green part of the spectrum have a slightly faster decay time compared to those giving rise to the blue part of the spectrum. It was not possible, however, to determine any predominant-level relaxation time from these data.

A simple model of a powdered polycrystalline electroluminescent sample may assume that the powder is made up of many independent single crystals, each one with a ground-, an intermediate-, and a trapping state. Electrons are excited to intermediate states from which they

Table 1

$\tau$   $\tau^0$  and  $\tau$  from square wave pulsing of a Sylvania "Nite-Lite" sample.  
exc decay

<u>Square Wave Repetition Rate</u>	<u><math>\tau^0</math></u>	<u><math>\tau_{exc}</math></u>	<u><math>\tau_{decay}</math></u>	<u>Filter</u>
100 cps	0	65 $\mu$ s	5.6 ms	None
250 cps	0	55 $\mu$ s	2.2 ms	None
1.0 Kc	6 $\mu$ s	50 $\mu$ s	0.52 ms	None
2.5 Kc	6 $\mu$ s	45 $\mu$ s	>100 $\mu$ s	None
4.0 Kc	5 $\mu$ s	45 $\mu$ s	> 40 $\mu$ s	None
6.0 Kc	8 $\mu$ s	40 $\mu$ s	> 25 $\mu$ s	None
250 cps	0	65 $\mu$ s	1.1 ms	Yellow
1.0 Kc	5 $\mu$ s	60 $\mu$ s	0.3 ms	Yellow
2.5 Kc	5 $\mu$ s	55 $\mu$ s	80 $\mu$ s	Yellow
250 cps	0	65 $\mu$ s	0.7 ms	Violet
1.0 Kc	0*	50 $\mu$ s	0.3 ms	Violet
2.5 Kc	5 $\mu$ s*	50 $\mu$ s	70 $\mu$ s	Violet

\* Peak existed, but very weak vs no filter and yellow filter case.



either recombine or are trapped. If the lifetime of the intermediate state is small compared to all other pertinent times, the rate at which the traps are filled is given by

$$\frac{dn(t)}{dt} = I(t) [N - n(t)] [1 - B] - DB(t)$$

where

$n(t)$  is the number of trapped electrons.

$N$  is the total number of traps.

$I(t)$  is the excitation rate of electrons into the traps.

$(1 - B)$  is the probability that an electron in an excited state is trapped.

$D$  is the rate of decrease of trapped electrons.

If  $L(t)$  is the total light output, then

$$L(t) = \int_v I(t) (1 - B) [N - n(t) - DBn(t)] dv.$$

In such an oversimplified picture, polycrystalline ZnS samples should have properties similar to ZnS single crystals. One would expect a smearing out of the single-crystal spectra and a small change in their predominant relaxation times, due to the perturbations present in polycrystalline substances. The results of the experiments are in qualitative agreement with such a picture.

A second commercial electroluminescent sample which was examined in great detail was the Sierraglow panel. Its dimensions were 2" x 2" x .2". It consisted of the usual three thin layers of metal, electroluminescent material, and a transparent conductor. These three layers were placed in between two plexiglass plates. For purposes of evaluation of the data, the specific heat of the sample was taken to be that of plexiglass. Several runs with and without applied voltages were made. In all cases the applied voltage was 370 volts peak to peak. The results are presented in Figs. 6 - 9. It is seen that both per-

inent parameters  $\alpha$  and  $\epsilon$  are very critically dependent on the "state" of the system. While  $\epsilon$  is almost a linearly increasing function of temperature for the case of no applied voltage, 400 cps. and 1 kcps., it is a linearly decreasing function of temperature at 4 kcps. This seems to indicate that the emission process might possibly take place with a characteristic time constant of some value between .25 and 1 millisecond. A similar change in slope is observed for  $\alpha$  between the no applied voltage and the 400 cps. case. For reasons of convenience the  $\alpha$ 's and  $\epsilon$ 's have been plotted for the various cases in Figures 10 and 11.

It is important to note that the final equilibrium temperature of both electroluminescent samples was much greater when a field was applied than under no-field conditions. A comparison of the final equilibrium temperatures is given in Table 2.

Table 2

<u>Sample</u>	<u>Applied Field Frequency*</u>	<u>Final Temperature</u>
Sierra Glow	None	220
Sierra Glow	400 cps.	275
Sierra Glow	4 kc.	303
Sylvania	None	243
Sylvania	60 cps.	293

\*The applied voltage was 110 volts rms. in all cases.

Only a small percentage of the difference in final equilibrium temperatures of applied-field to no-field runs can be attributed to Joule heating. There, apparently, is an intrinsic property of the material which causes most of the temperature difference.

(FIG. 1)

300°K  
60 CYCLES

RELATIVE INTENSITY

WAVELENGTH (Å)

5520

4995

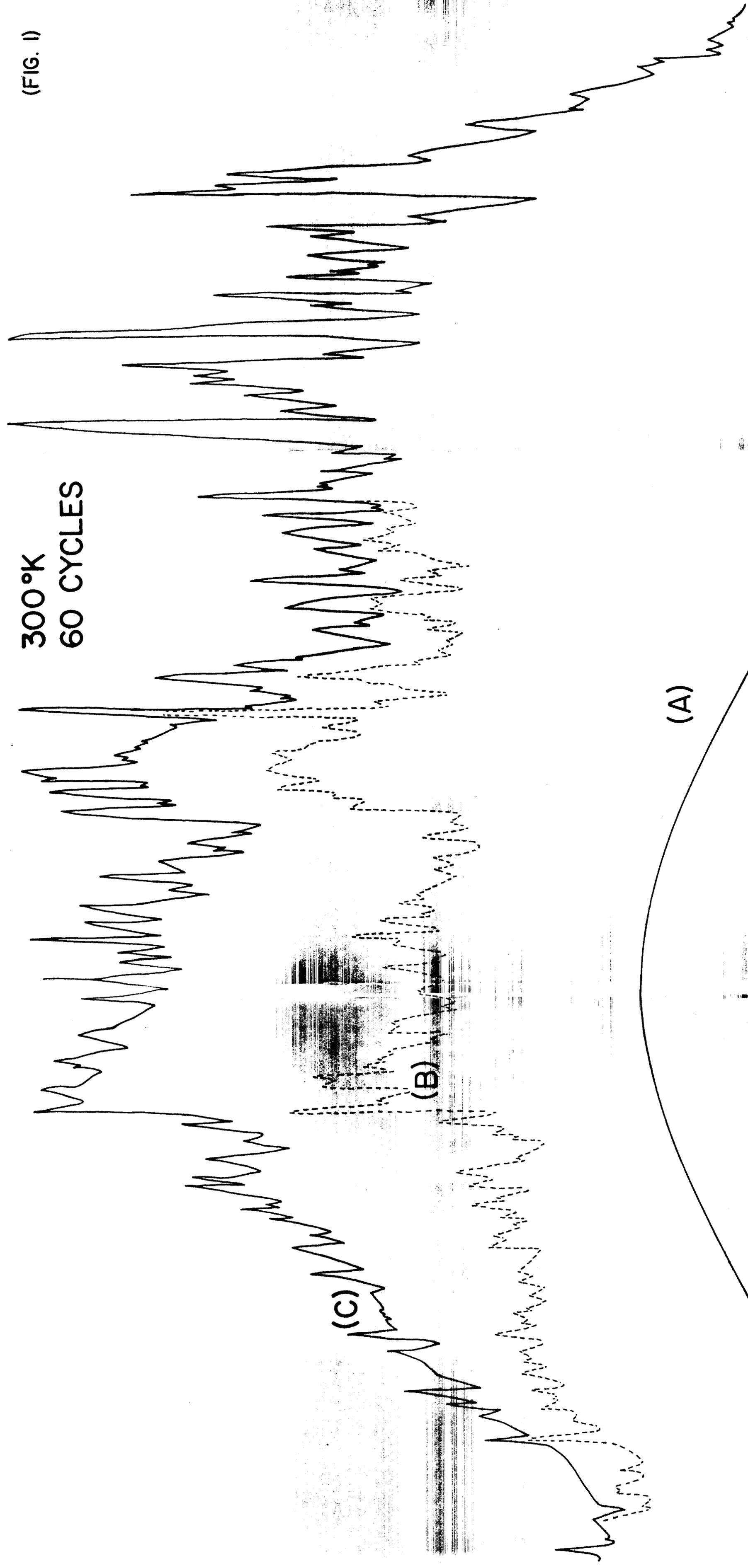
4470

3945

(A)

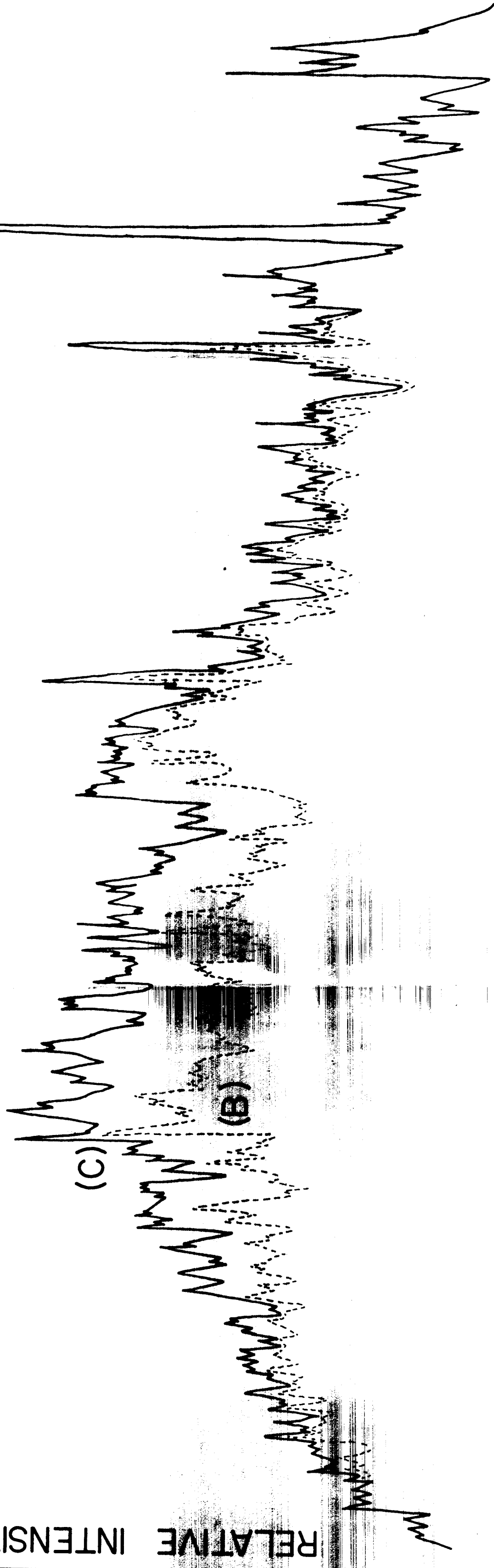
(B)

(C)

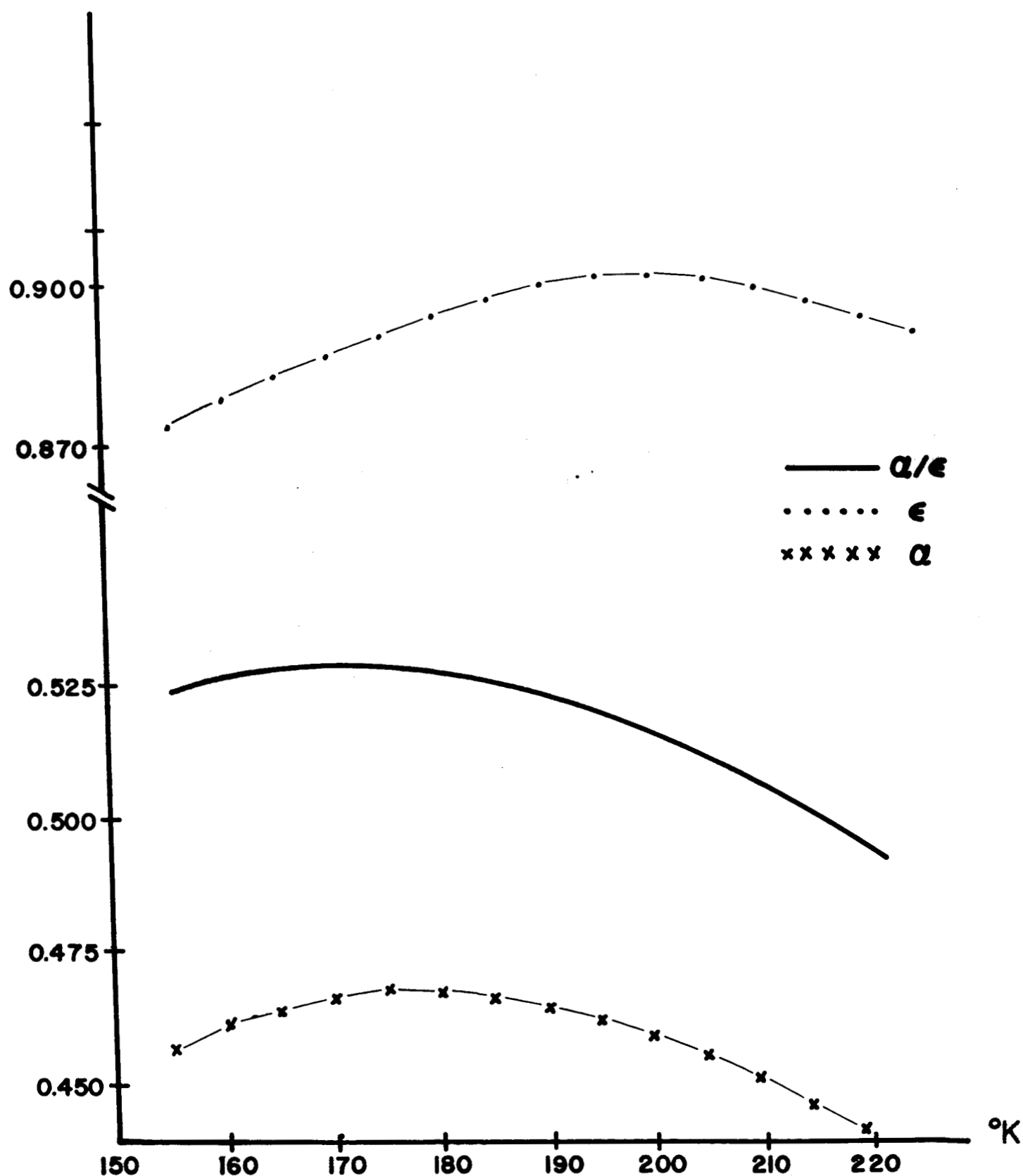


100 °K  
60 CYCLES

RELATIVE INTENSITY

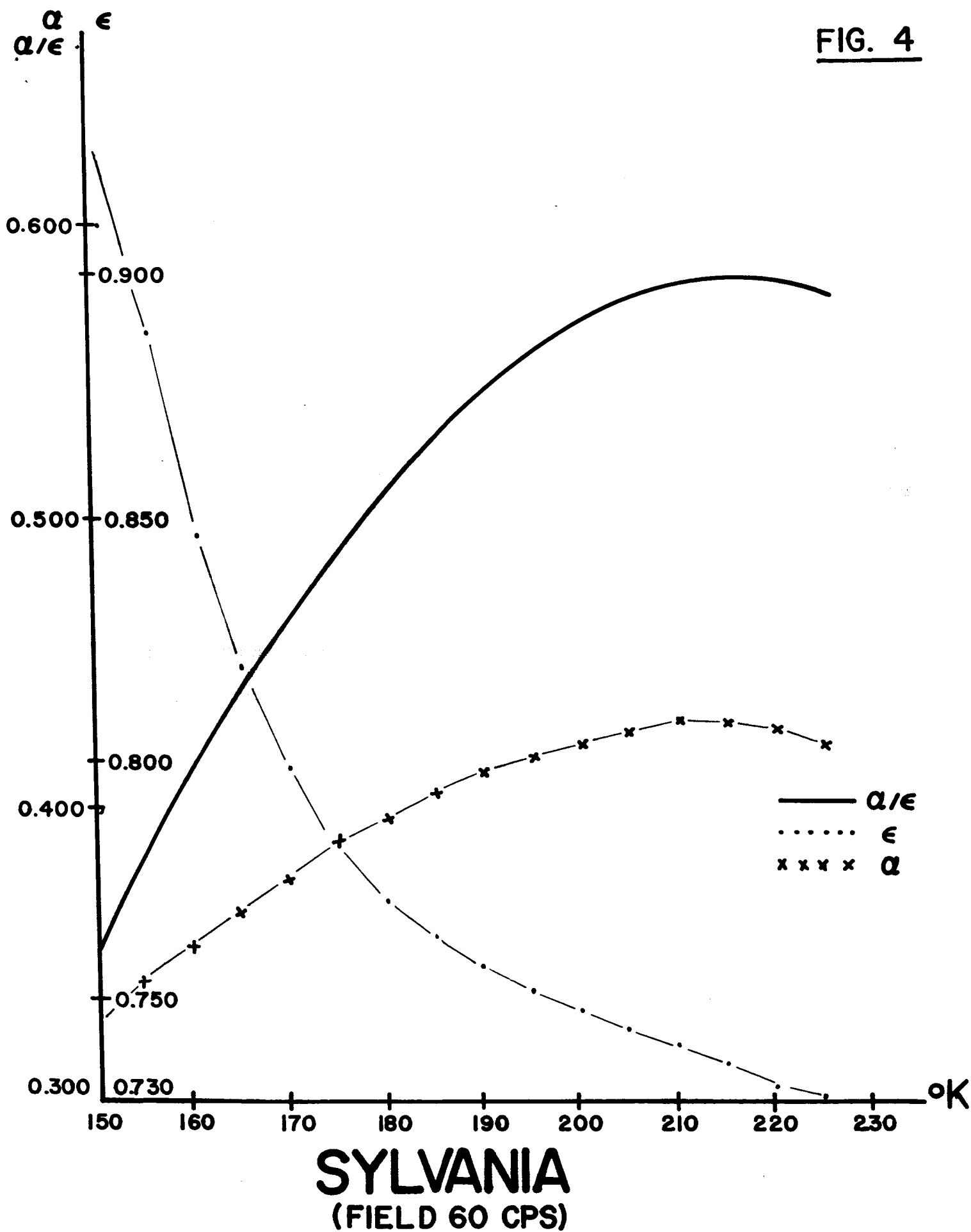


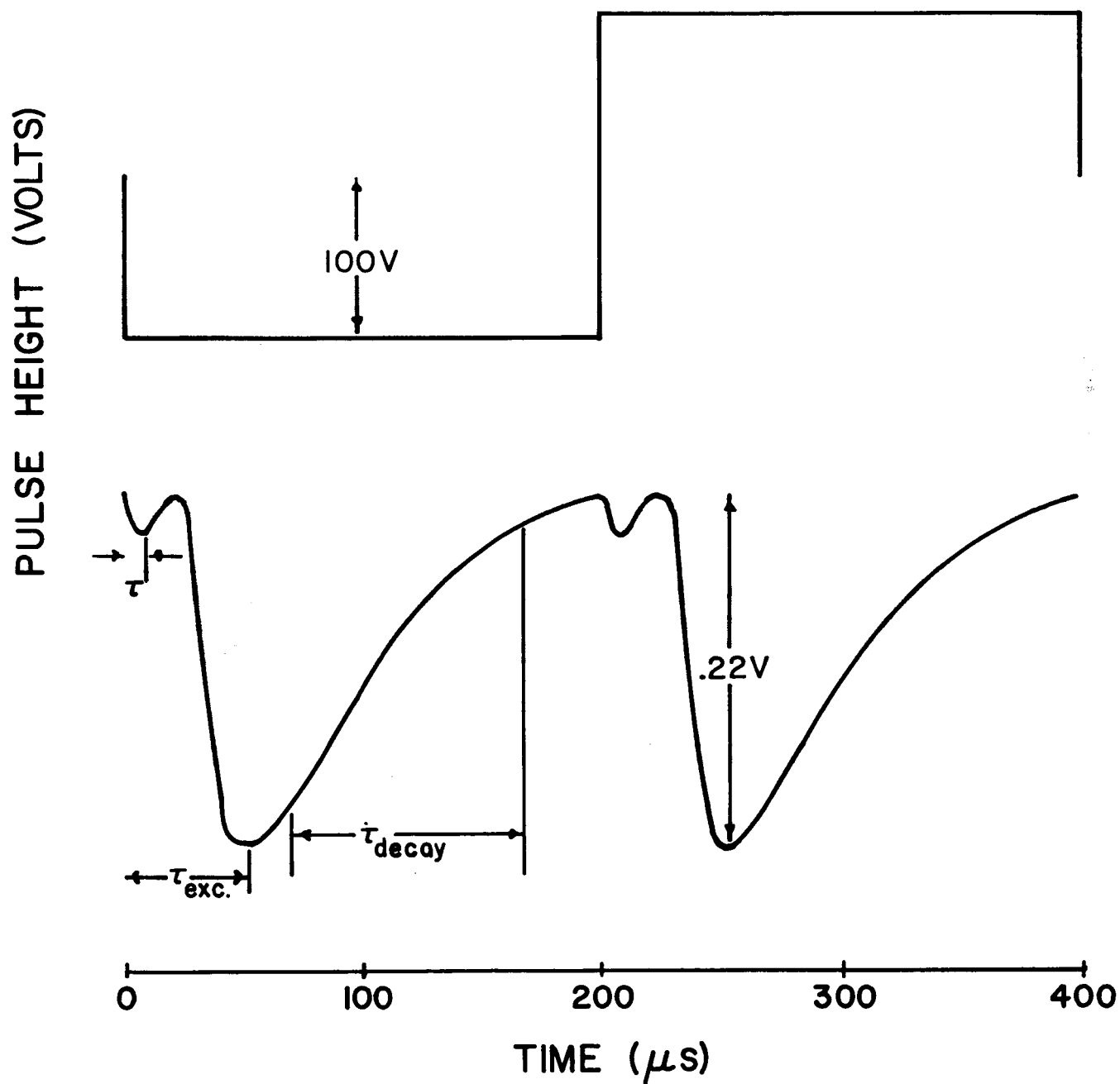
**FIG. 3**



**SYLVANIA**  
(NO FIELD)

FIG. 4





OSCILLOSCOPE PATTERN OF EXCITATION PULSE (TOP) AND LIGHT OUTPUT (BOTTOM) FOR SYLVANIA "NITE - LIGHT" SAMPLE.

**FIG. 6**

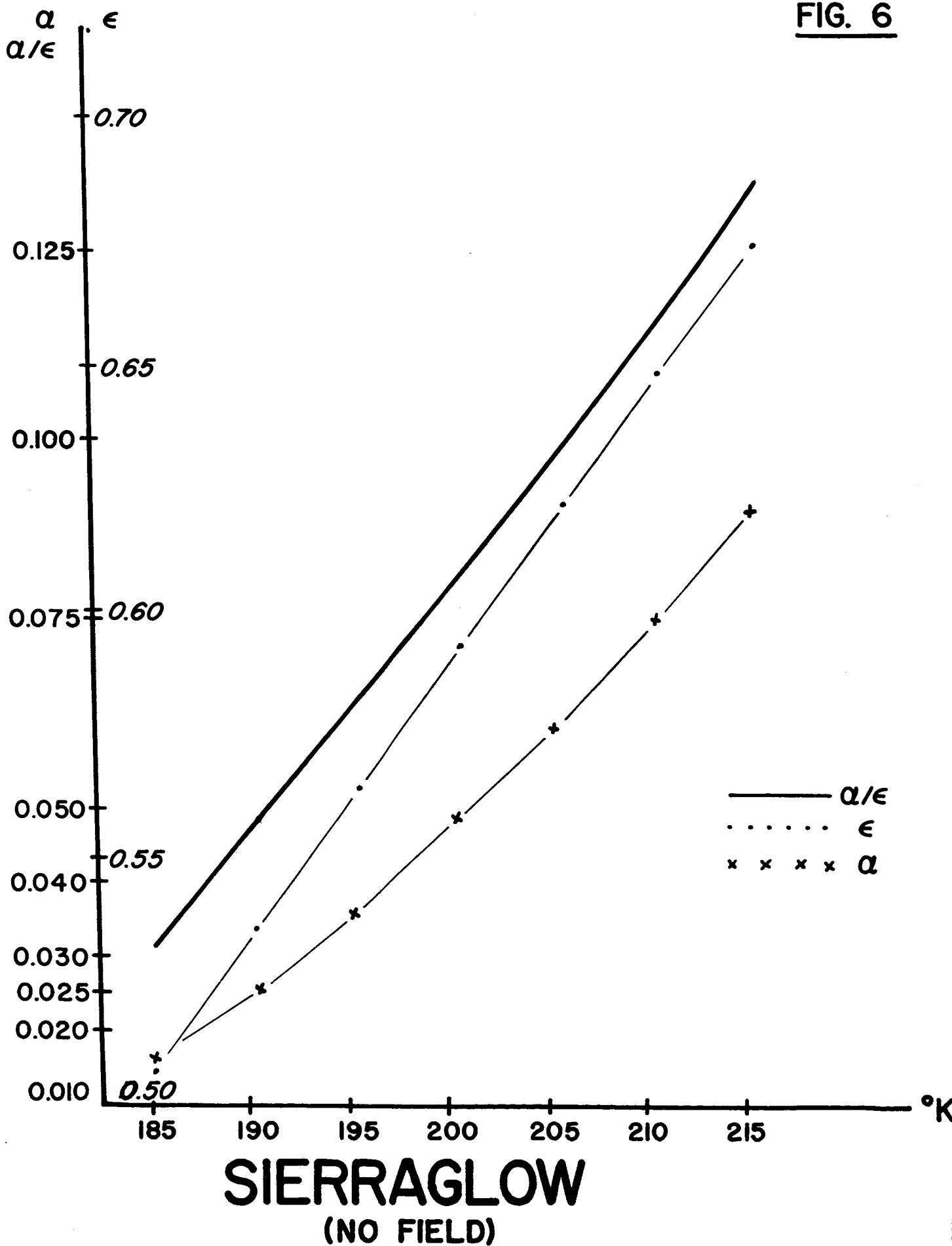




FIG. 7

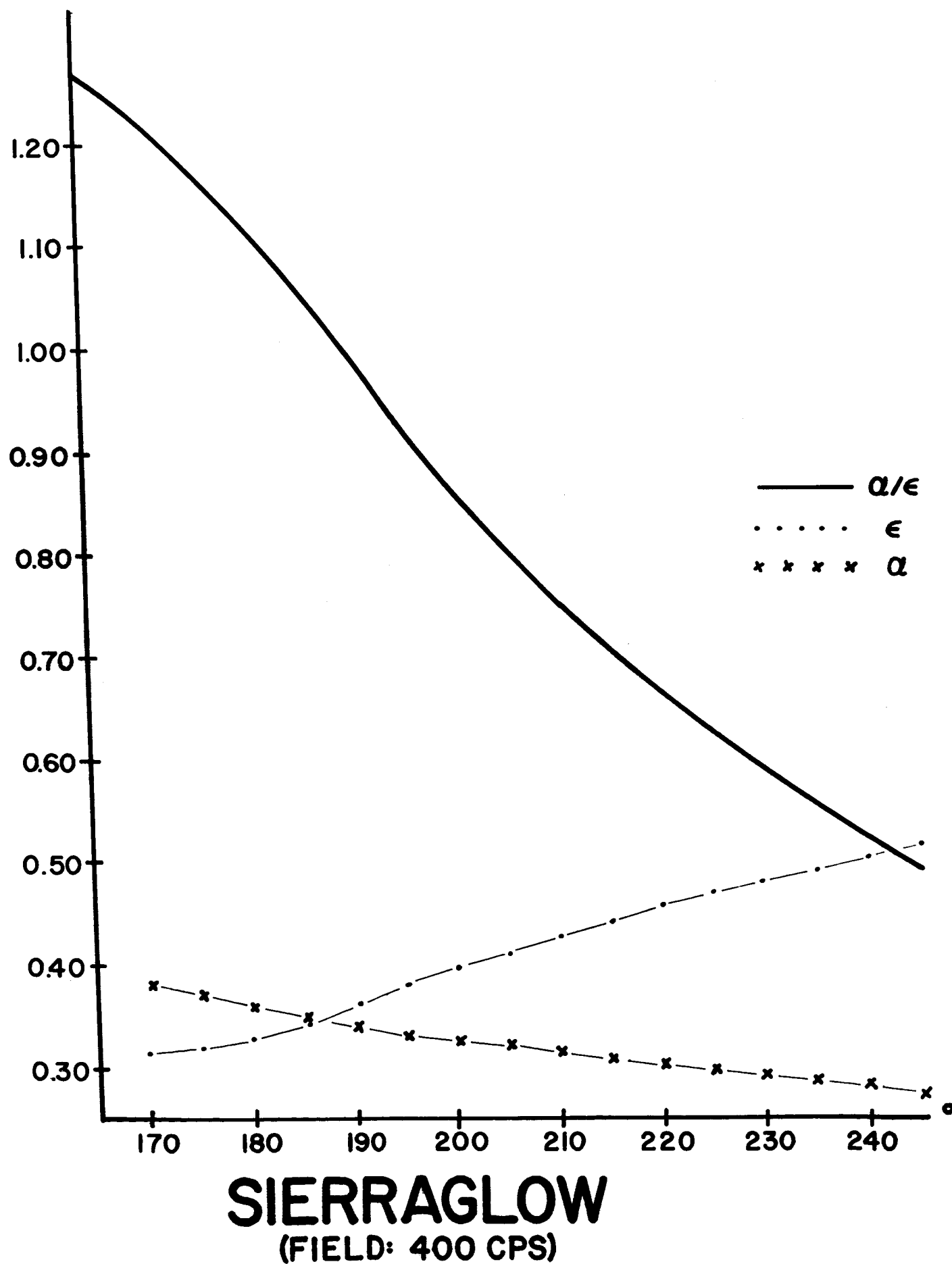
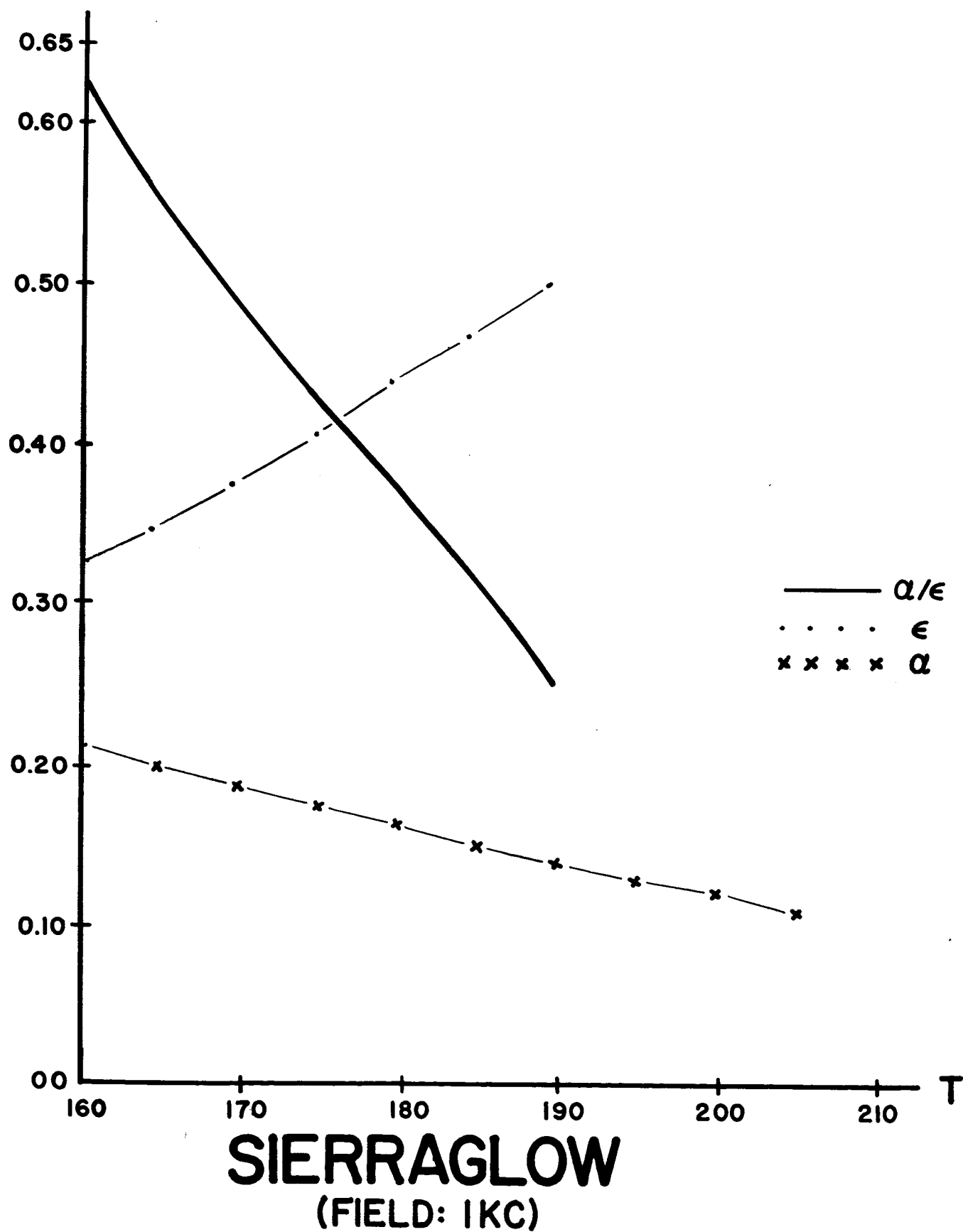
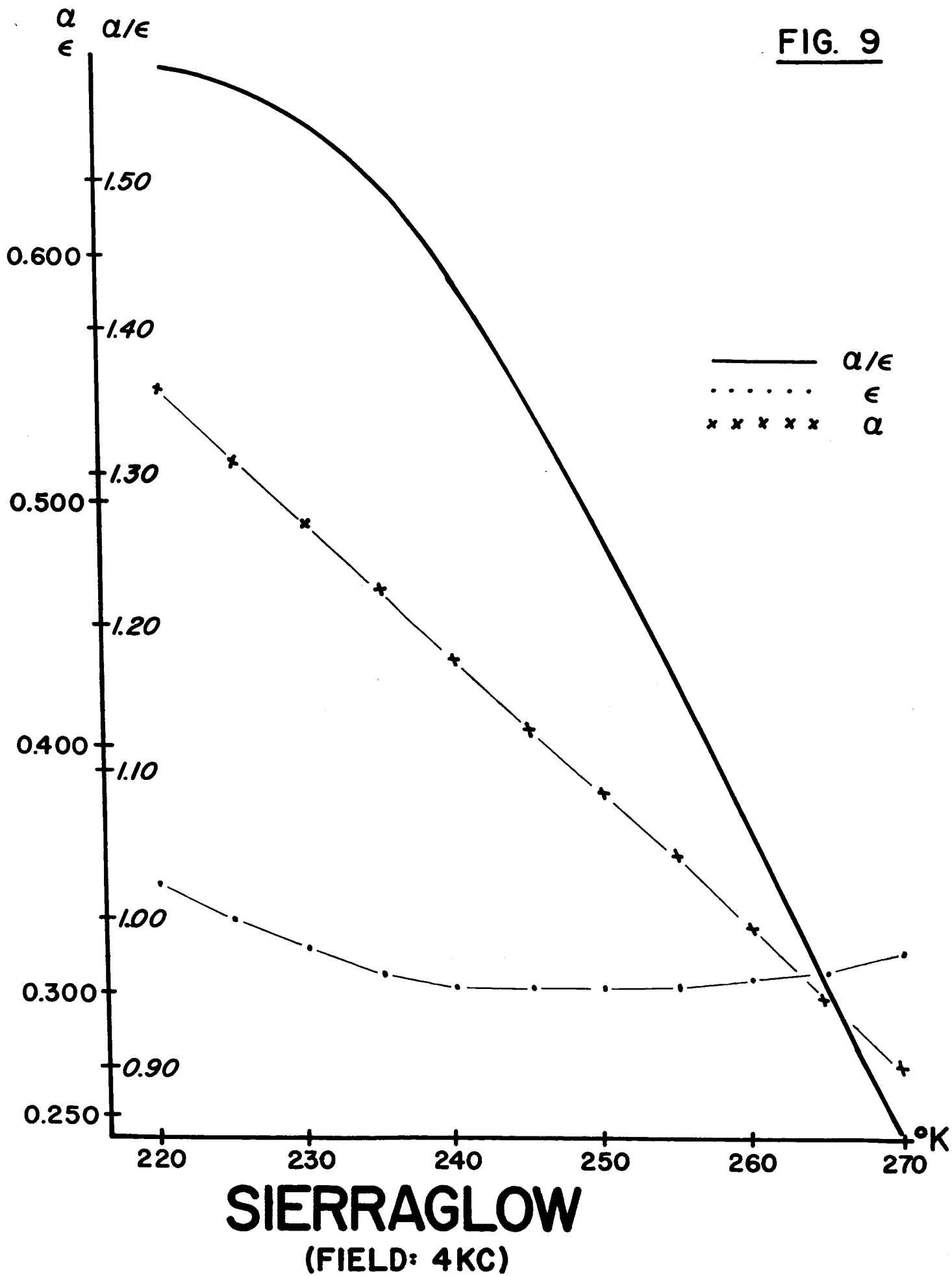


FIG. 8



**FIG. 9**



**FIG. 10**

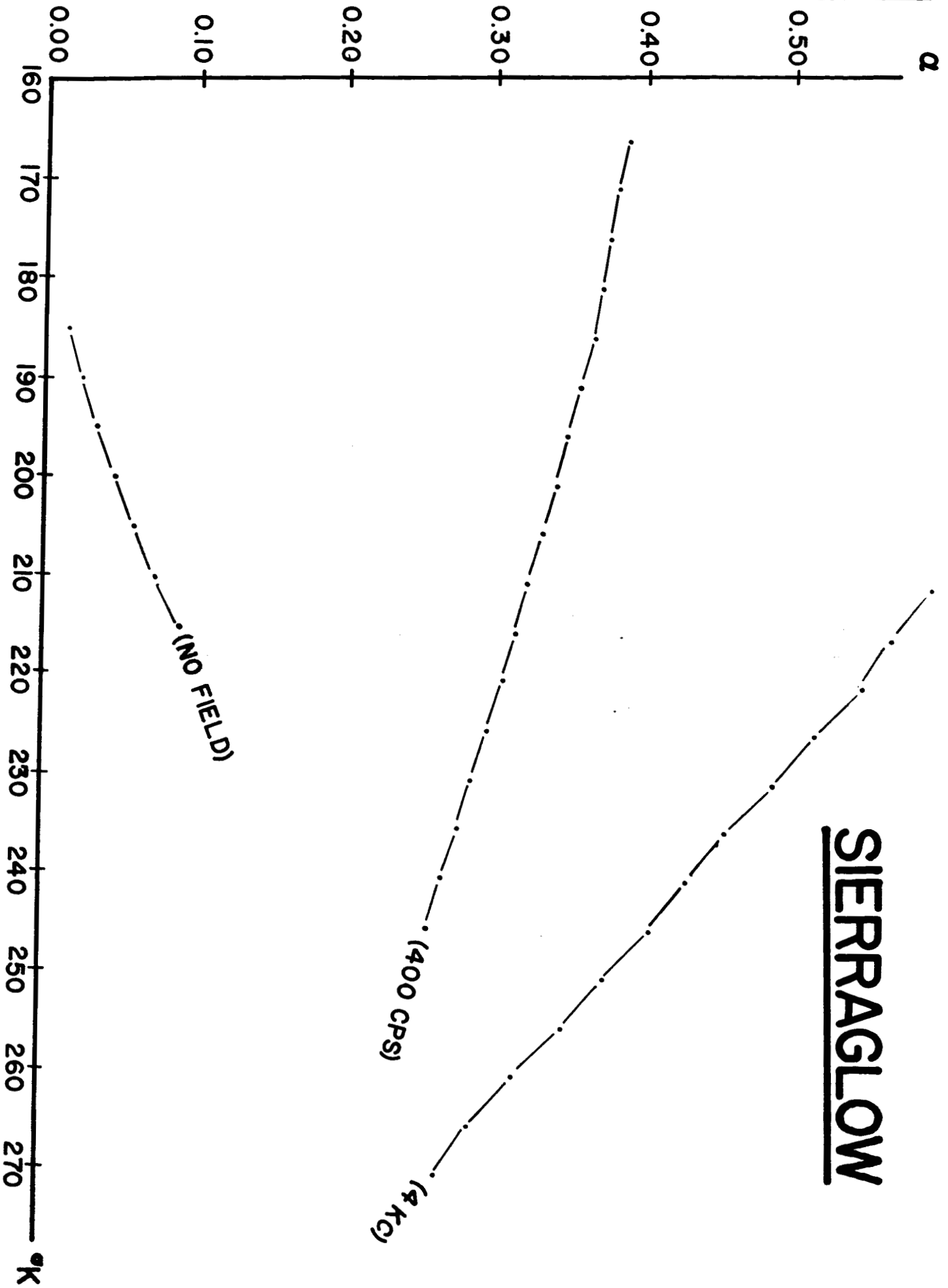
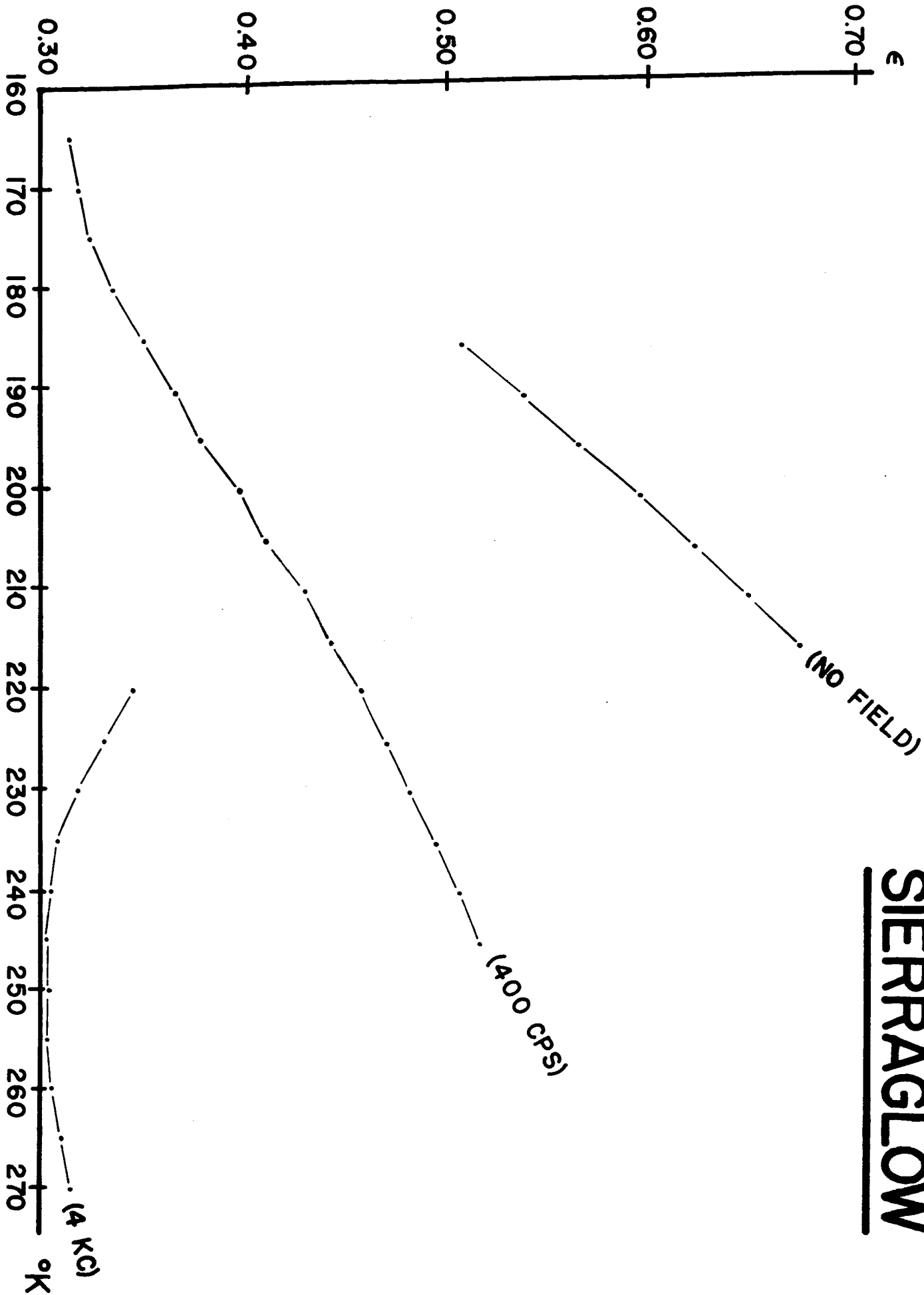


FIG. II

# SIERRAGLOW



# Emittance and Absorptance of Ni-Ba Cells

A brief study of  $\alpha$ ,  $\epsilon$ , and  $\alpha/\epsilon$  was undertaken for a photovoltaic cell. Assuming that the optical properties of a given surface layer are mainly determined by the number of free electrons per unit, volume one would expect drastic changes in these parameters only when the number of free electrons is changed appreciably. It was desirable to obtain immediate results to ascertain whether or not this line of investigation would hold any promise whatsoever. For this reason two materials that lend themselves easily to the preparation of photovoltaic cells were chosen. These were Ni and Ba with electronic work functions of 5.01 eV and 2.50 eV, respectively. The difference in work functions corresponds to a wavelength of about 4950A.

The cells were prepared in a vacuum chamber by evaporating the barium metal coatings onto a Ni substrate. The samples were then removed and placed in a nitrogen filled dry box where electrical contacts were made by soldering one wire onto the backsurface of the sample and another one to a washer which was screwed onto the Ba-layer. Since this layer was only about 600 Å thick, soldering could not be done, while conducting paints, which are at times used for glueing, do not guarantee proper contacts.

The thickness of the Ba-layer was determined from the geometry in the vacuum system and the amount of evaporated Ba. In order to estimate the amount of radiation which penetrates to the boundary layer, the equation

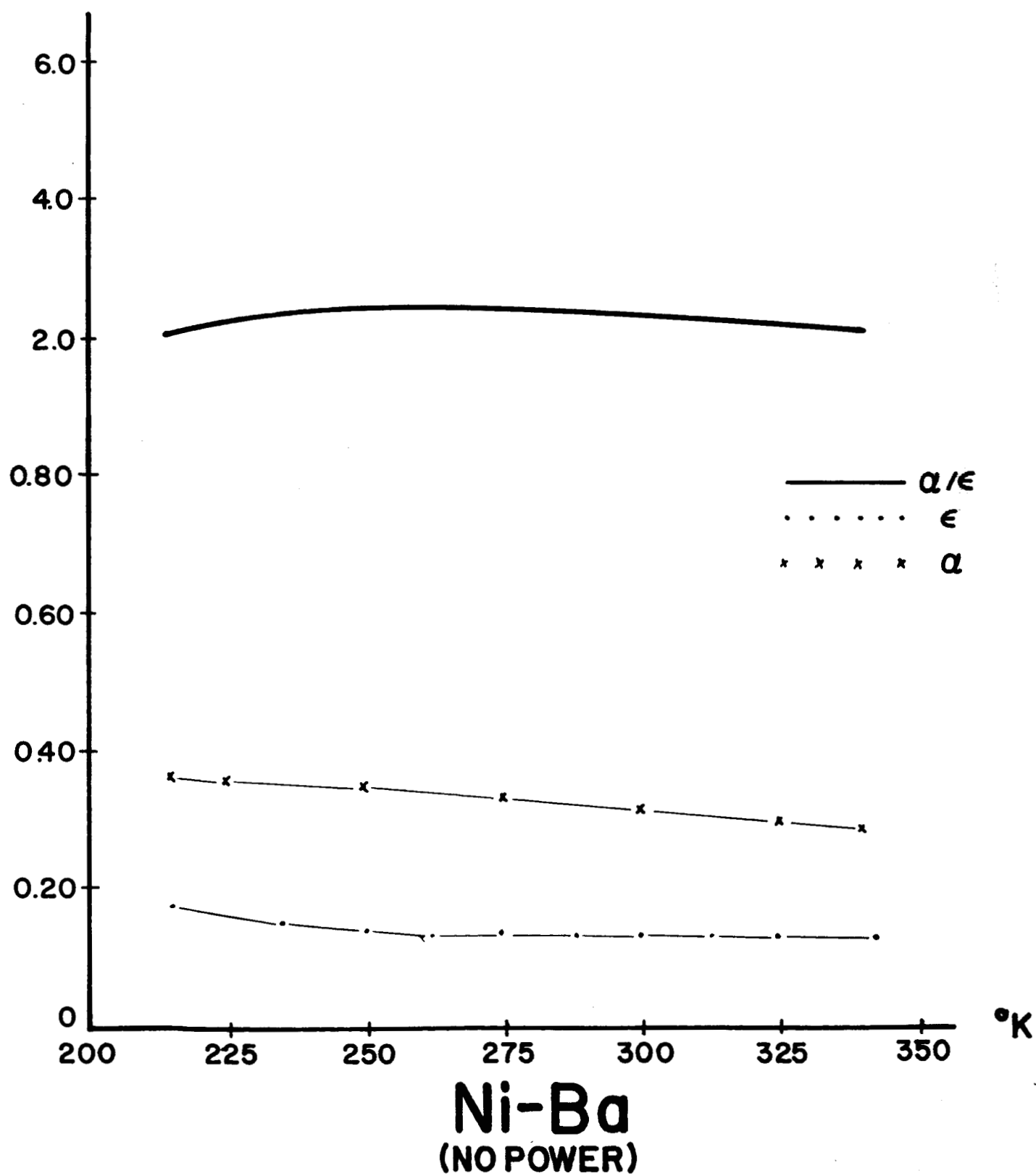
$$\delta = \sqrt{\frac{\lambda}{\pi c \mu \sigma}}$$

for the skin depth was used. Here  $\lambda$  is the wavelength;  $c$  the velocity of light;  $\mu$  the permeability of free space; and  $\sigma$  the conductivity of Ba. For a wavelength of 5000 Å one obtains a penetration

depth of  $160 \text{ \AA}$ . This means that about 2% of the incident radiation does penetrate to the Ni-Ba junction.

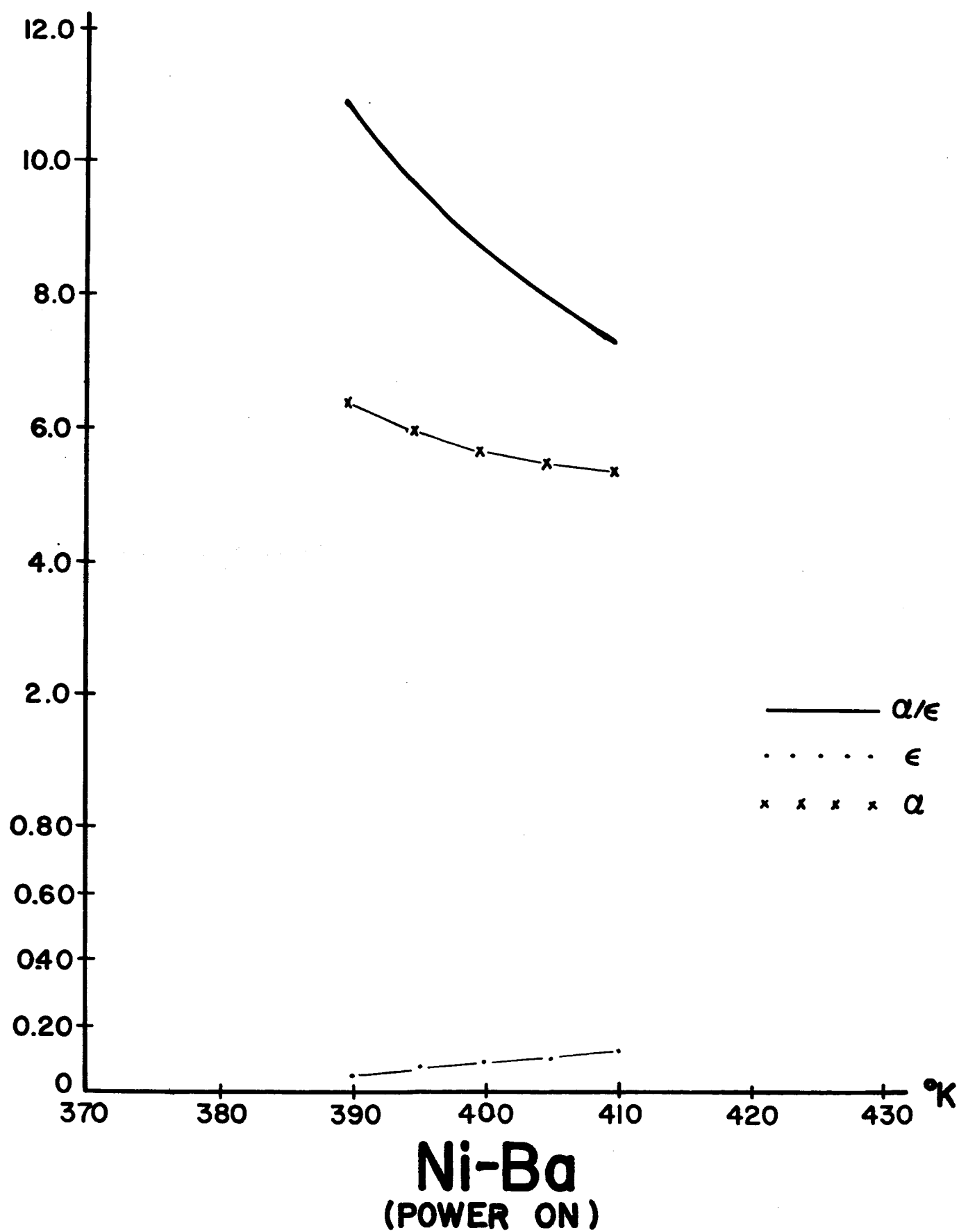
Altogether four runs were made with this sample. One with the full intensity of the S. S. beam and one with three quarter intensity. These runs were repeated with the addition of putting 2 volts across the sample which had a resistance of about  $0.5 \Omega$ . The results are presented in Figs. 12 and 13. As can be seen, neither  $\alpha$  nor  $\epsilon$ , nor their ratio, change appreciably over a temperature range of  $200^\circ \text{K}$  as long as no electrical power is put into the sample. The upper limit of the temperature is given by the power input available from the solar simulator. Comparison of these three curves with the ones obtained by putting 2 volts across the sample demonstrates a striking difference. While  $\epsilon$  decreases by a factor of 2 with an additional change in slope,  $\alpha$  increases by a factor of 2 with a similar large change in slope. At the present time it is not possible to explain these results. It is, however, obvious that the emission process is a great deal less influenced by the presence of moving charges than the absorption process. An explanation can possibly be given after a number of pertinent parameters in the experimental setup have been altered in a systematic way. This was not done in the present study because an appreciable change in  $\alpha$  and  $\alpha/\epsilon$  can only be obtained with the help of an unacceptable large electrical power input.

FIG. 12





**FIG. 13**



### Emittance and Absorptance of Cu-PbS Cells

In order to check the temperature dependence of  $\alpha/\epsilon$  for a photo conducting layer, a Cu-PbS cell was prepared in exactly the same fashion as described in the previous section for the Ni-Ba samples. The lead sulphide used for these cells was prepared in the laboratory on the basis of information given elsewhere<sup>(6)</sup>. Briefly, the material was produced by passing  $H_2S$  gas through a solution of lead acetate. PbS precipitated and was filtered out. It was then deposited on a substrate of copper.

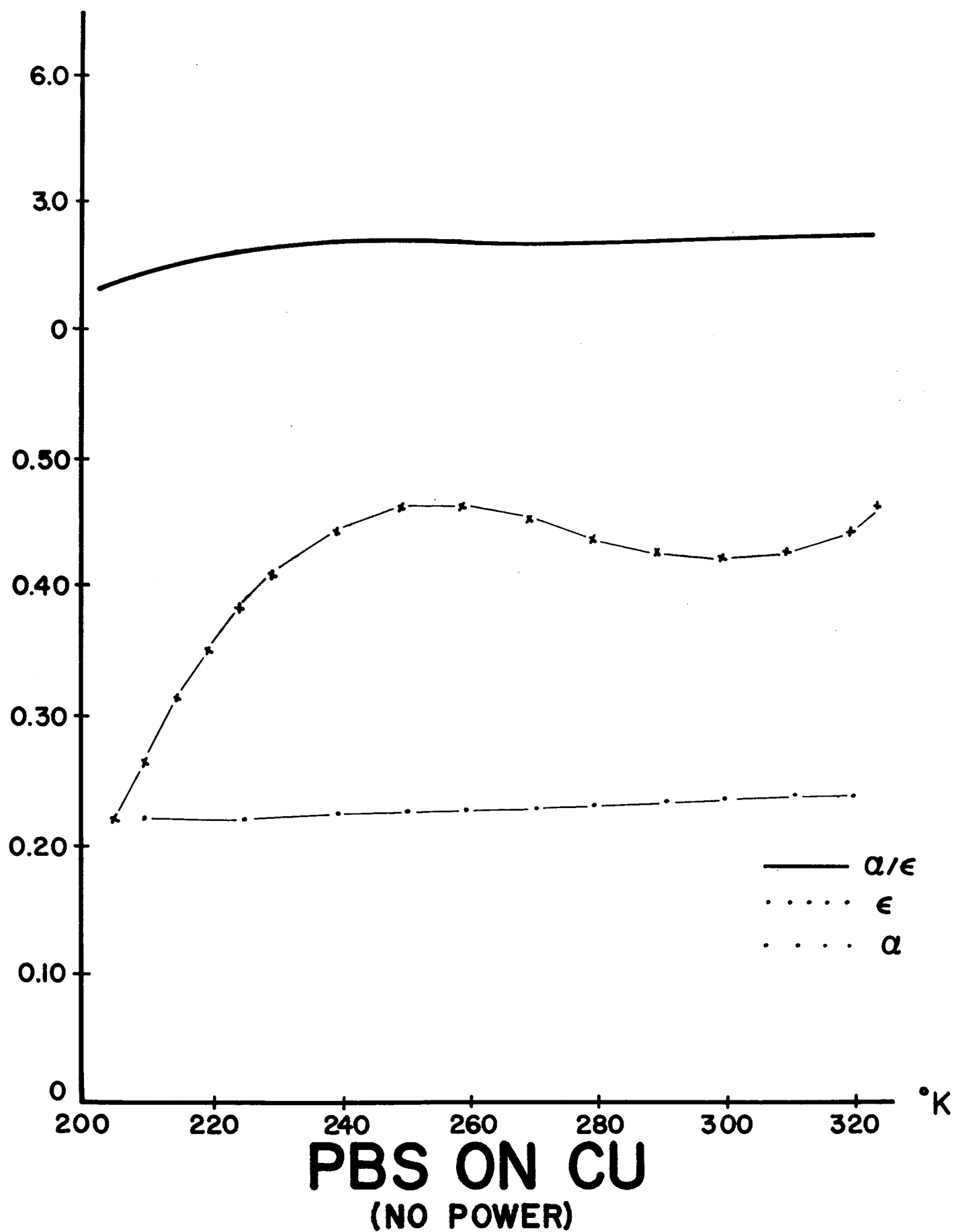
Using again the equation for the skin depth, one obtains a value  $\delta = 1000 \text{ \AA}$  for a temperature of  $300^\circ \text{K}$ . The value of the conductivity was  $25 \text{ mho/cm}$ <sup>(7)</sup>. This number gives the right value for  $\rho$  to within 10% as long as the temperature does not vary beyond the range from  $200^\circ \text{K}$  to  $400^\circ \text{K}$ . Published values of the absorption co-efficient<sup>(7)</sup> yield very much larger penetration depths so that for a PbS layer of about  $500 \text{ \AA}$ , as was used in this experiment, the incident light pervaded the whole layer of PbS without appreciable decrease in intensity throughout.

Again four runs were made; i.e., one with the full intensity solar simulator beam, one with three quarter intensity with and without an applied 2 volts across the sample. The results are presented in Figs. 14 and 15.

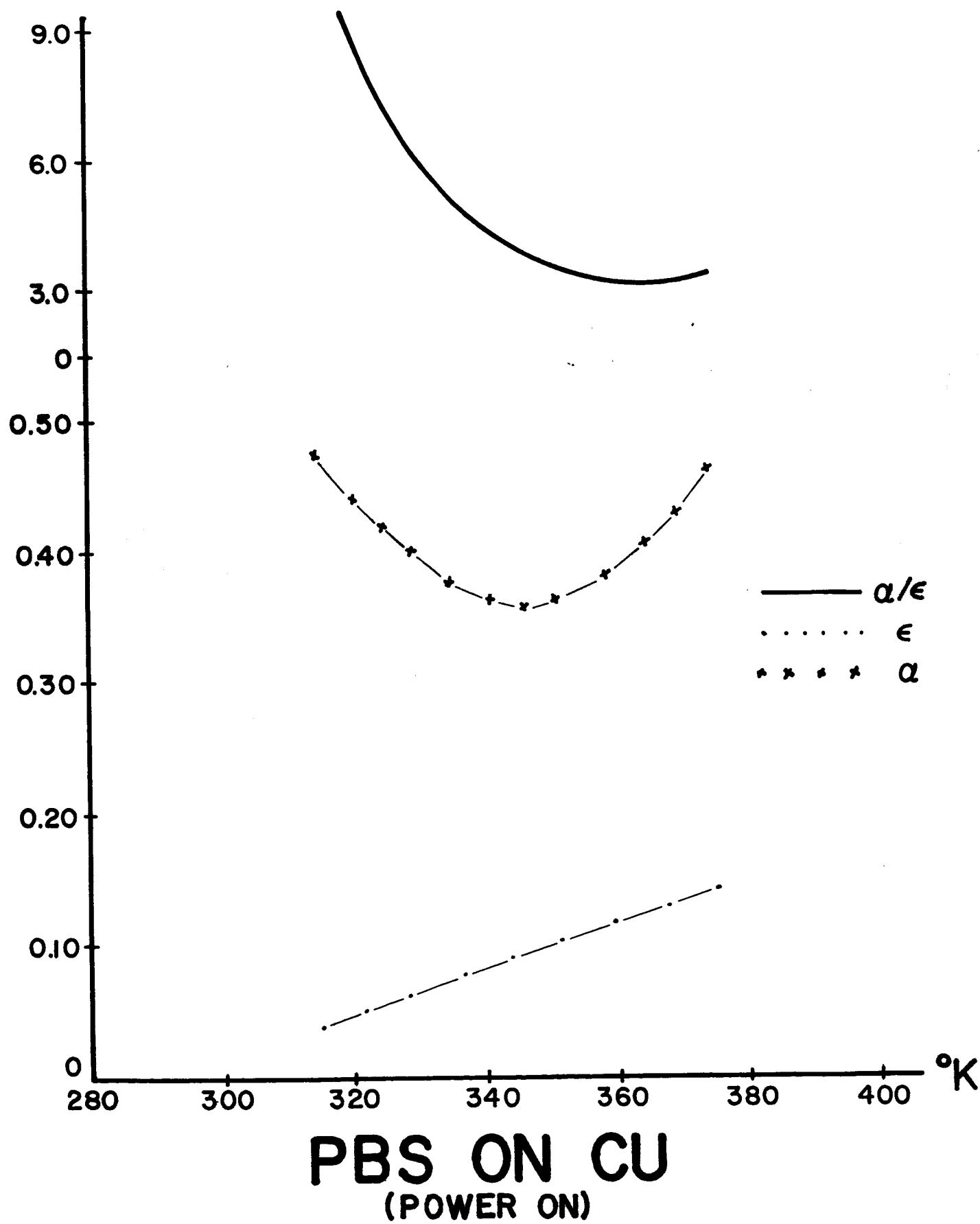
One obtains, without electrical power input, an almost constant value for  $\epsilon$  while  $\alpha$  changes rapidly from  $200^\circ \text{K}$  to  $240^\circ \text{K}$  and remains reasonably constant thereafter. As two volts are put across the sample, one finds no appreciable change in  $\epsilon$  while  $\alpha$  decreases by a factor of almost 5. The slopes in the curves change drastically but the important conclusion is again that no practical application for this project can be obtained by the use of PbS-Cu cells since only with a large power drain does it seem to be possible to change  $\alpha/\epsilon$  by any significant amount.

The underlying physical reasons for the behavior of  $\alpha$ ,  $\tau$ , and  $\alpha/\epsilon$  have not been worked out. However, it is believed that a great deal can be learned about the basic physics by varying the conductivity of the PbS layer and performing a variety of experiments with different ratios of light and electrical power input.

**FIG. 14**



**FIG. 15**



### Absorptance and Emittance of Germanium

A brief study of the emittance and absorptance of a typical semiconductor, an n-type sample of Germanium with  $10^{16}$  donors per cm<sup>3</sup>, was also made. Figs. 16-18 represent the obtained results. Again  $\alpha$  and  $\epsilon$  do not change much for the case of no applied voltage between 155° K and 265° K. A similar situation exists for the case of an applied voltage giving rise to a current of 75 ma, although there appears to be a slight rise of the emittance with temperature. On the other hand the emittance changes by a factor of about 2.5 over a temperature range from 300° K to 360° K with a current input of 125 ma. There may be two reasons for this effect. First it is in principle possible that the thermal emittance of Germanium actually increases, but second it could also be that  $\epsilon$  appears to increase because of the additional emission of infrared radiation corresponding to the energy gap between the conduction and valence bands. In addition it should be noted that the interaction between the moving charges and the incident solar radiation may allow transitions for which the crystal momentum is not conserved. This in turn would either split the emission line or broaden it, depending on the details of the interaction. The absorptances on the other hand remain fairly constant within the assigned errors. This indicated that for a given condition, i.e., for a given current the number of available states into which electrons can be transferred during the absorption process do not change appreciably.

Work with this type of sample was not pursued any further because it was found again that any worthwhile change in the  $\alpha/\epsilon$  ratio could only be obtained for unacceptable high currents.

**FIG. 16**

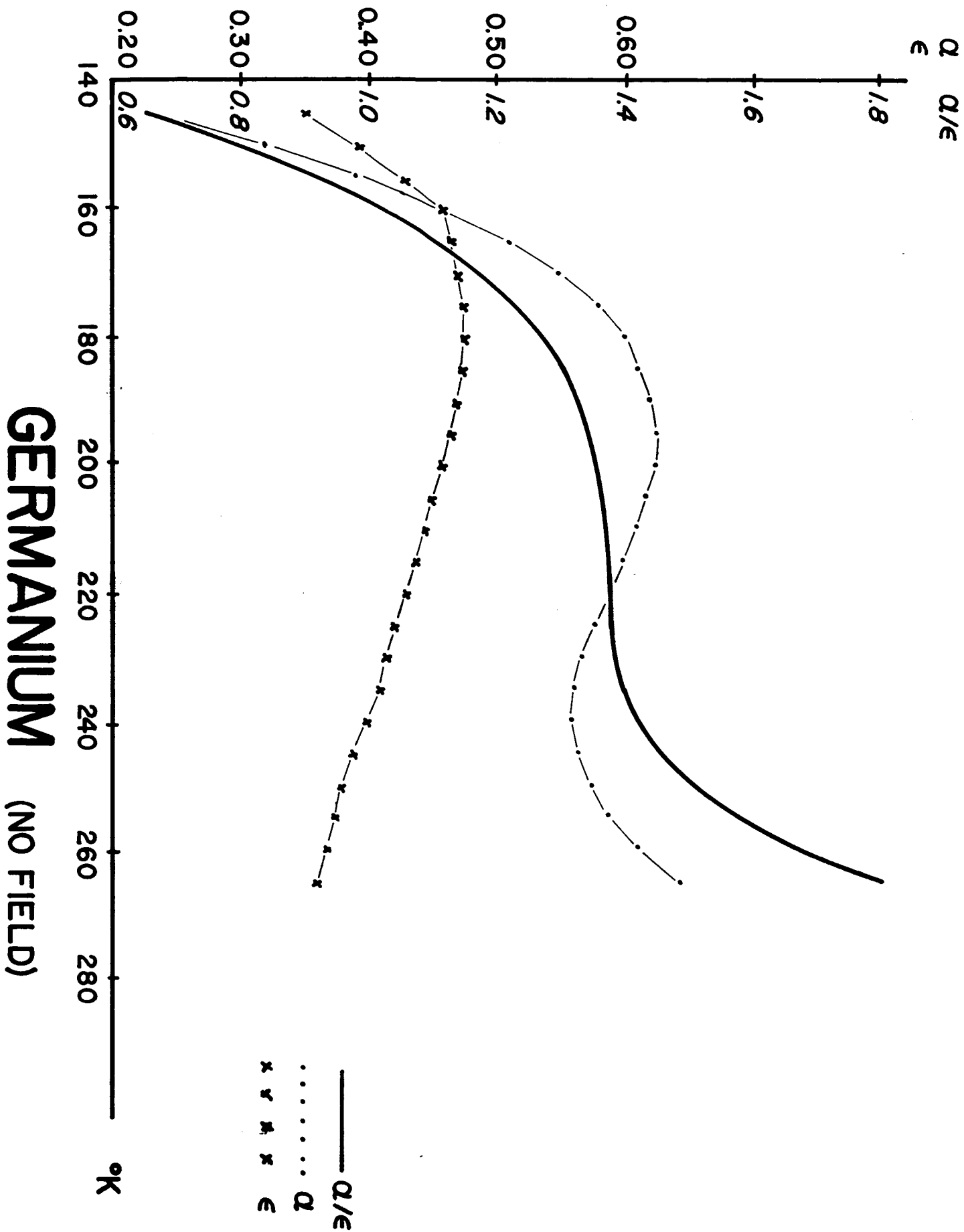
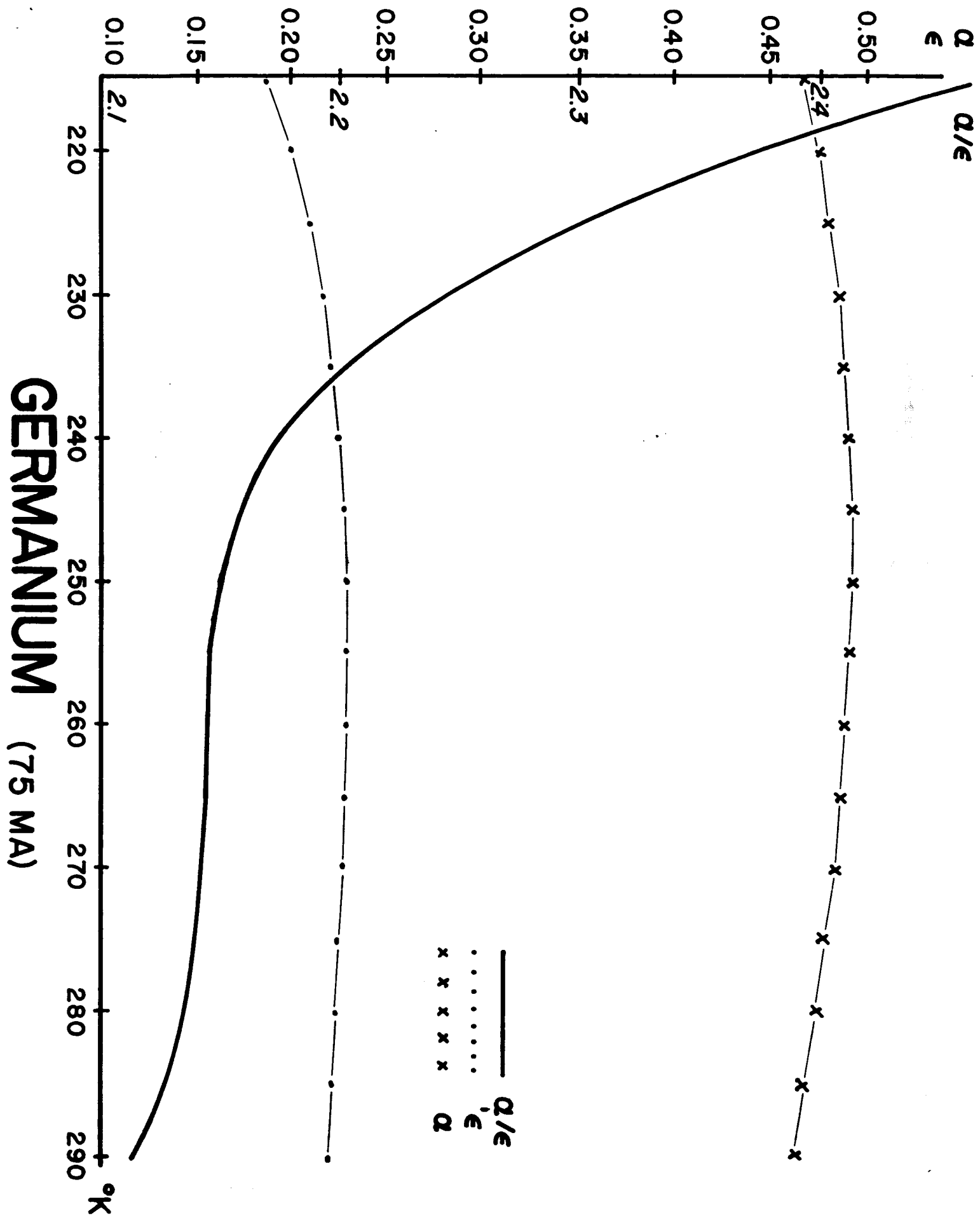
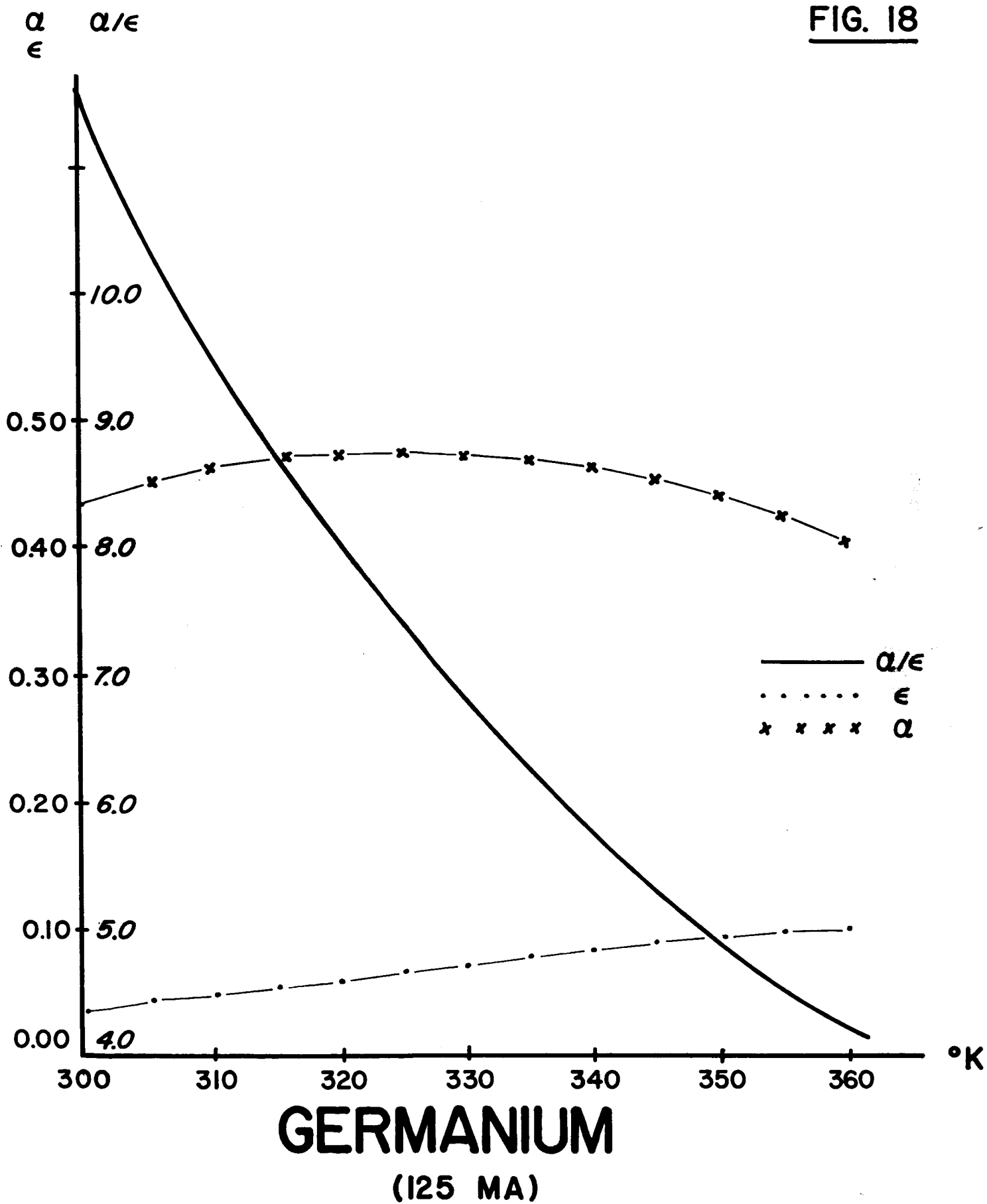


FIG. 17





**FIG. 18**



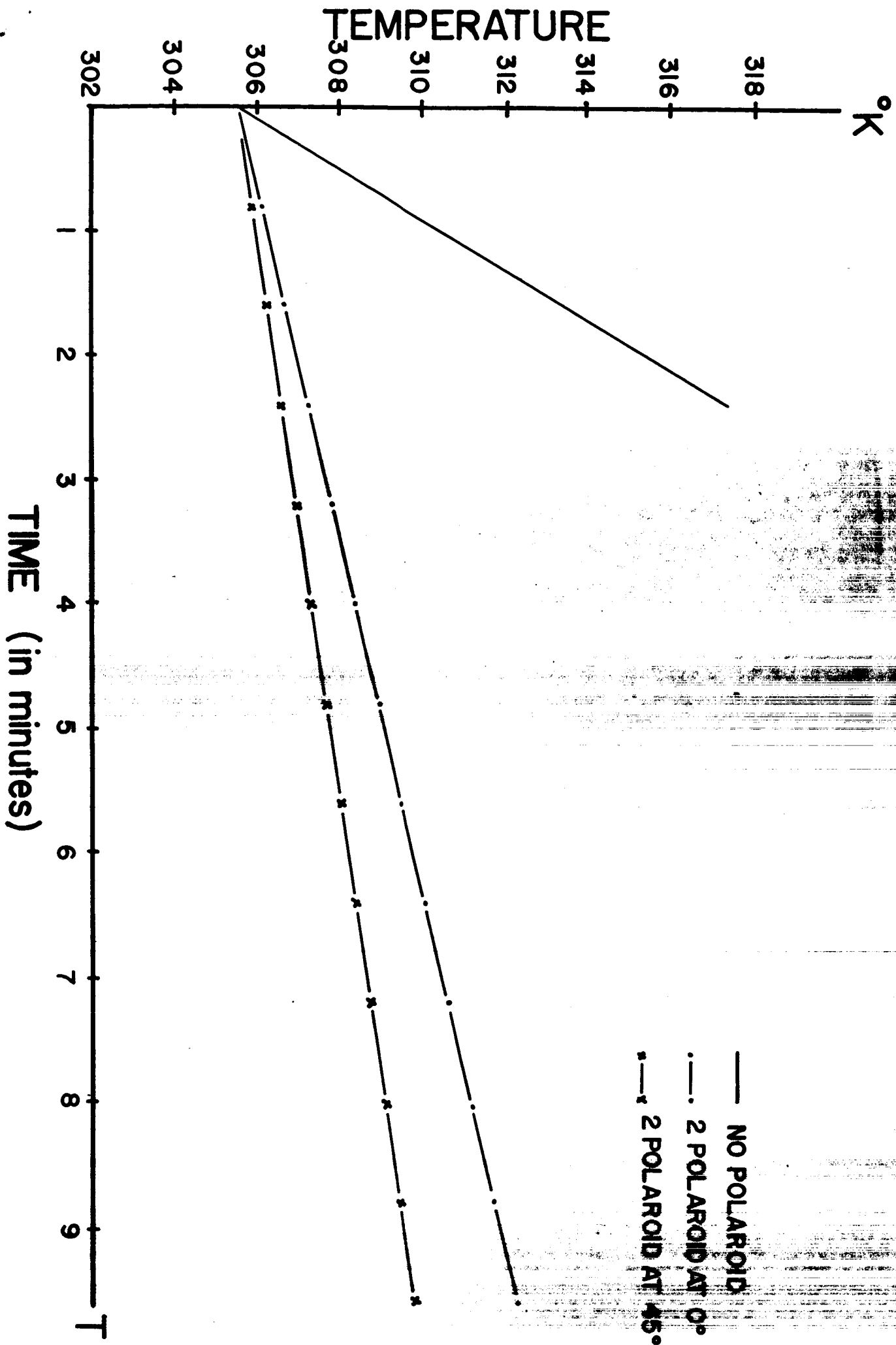
## Electro-optic Effect

It appears possible to control the equilibrium temperature of a surface by adjusting the magnitude of the E-vector of the solar radiation incident on the surface. This would require some type of material between the surface and the radiation source whose polarization properties can be adjusted. In order to test this possibility, the simplest material to use is polaroid sheet. In the experiment, a copper disc, two inches in diameter, with a thermocouple silver-soldered on the back surface, was suspended in the space simulation chamber in the usual manner. A calibration run was made to determine the rate of rise of the temperature with time for the sample alone. Two sheets of polaroid were then suspended in the beam in front of the sample with the planes of polarization making an angle of zero degrees (maximum transmission), and the heating run repeated. The planes of polarization were then set at  $45^\circ$  to each other and another heating run taken.

The results are shown in Figure 1 for these three cases. It is obviously possible to control the equilibrium temperature of the sample by adjusting the polarization angle.

In order to determine whether the polarization produced by the polaroid resulted in a temperature rise of the polaroid material itself, runs were made with a thermocouple sandwiched in between the polaroids but not in direct beam from the solar simulator. A negligible temperature rise was observed. Measurements of the light reflected from the polaroids were made and it appears that most of the polarization properties of polaroid are due to reflection of the non-transmitted E-vectors in the unpolarized incident radiation.

Since mechanical adjustment of polarizers is not very attractive in the final system, investigations are in the planning stage to study the use of electro-optic crystals which require only a DC voltage to effect the polarization change.



### Bibliography

1. W. J. Thaler, E. J. Finn, P. A. Treado, J. Nakhleh, Journal of Applied Optics (to be published)
2. E. R. G. Eckert, Introduction to the Transfer of Heat and Mass, McGraw - Hill Book Co., Inc., New York (1950), p. 375
3. Matossi, Nudelman, "Some Electroluminescent Effects and Their Interpretation", paper presented at Spring 1954 meeting of The Electrochemical Society
- Birman, "A Theory of the Electroluminescence of ZnS Single Crystals", paper given at Spring 1956 meeting of the Electrochemical Society
5. Frankl, "Further Experiments on Electroluminescence of ZnS Single Crystals", paper given at Spring 1956 meeting of the Electrochemical Society
6. Smith, Jones, Chasmas, The Detection and Measurement of Infrared Radiation, Oxford University Press, 1957, p. 171
7. Scanlon, Solid State Physics, Academic Press Inc., Vol. 9, 1959

**THEORETICAL SECTION**

Superposition Formulas for the Temperature Gradient Problems  
Arising in the Measurement of Absorptance and Emittance

1. Introduction

In the Gordon-type of measurement<sup>1,2</sup> of the absorptance and emittance of metals, the sample being measured may be employed in sufficiently thin layers so that temperature gradient effects may be neglected. The determination of absorptance and emittance from heating and cooling data is then governed by a first-order ordinary differential equation, and the major mathematical difficulty arises only in connection with the numerical analytic techniques needed in correcting for random and systematic errors in the measurement.

In the case of electroluminescent materials, however, for reasons of support and electrode insulation, the electroluminescent package must contain one or more layers of dielectrics, and experiment has shown that because of their presence a sizeable temperature gradient exists across the package during the heating and cooling processes. The problem of determining the absorptance and emittance of the embedded electroluminescent material from front and back temperatures then takes on an entirely more complicated complexion, and now requires a more extensive use of the full mathematical apparatus of heat transfer theory.

The simplest case of this temperature gradient problem is that in which (a) absorption of radiation (and hence emission) by the dielectric layers may be neglected, and (b) the specific heats and thermal conductivities of the materials involved may be considered temperature-independent over the temperature range of the measurement. In this simplest case an analysis, to be presented in a subsequent report, shows that, by means of a Green's function approach, the problem of determining the absorptance  $\alpha(T)$  and the emittance  $\epsilon(T)$  as functions of absolute temperature  $T$  may be formulated as a pair of simultaneous linear integral equations in these two unknowns. Moreover, even when the above conditions (a) and (b)

cannot be assumed, the Green's function approach can be expected to play a fundamental role.

One recalls the general philosophy behind the Green's function approach to the solution of a given class of boundary-value problems on a linear operator<sup>3</sup>. Namely, rather than solve each problem of this class on an individual basis, one instead devotes his sole effort to solving a certain singular problem of this class, the solution of this singular problem being called the Green's function for the class. The solution of any other problem of the class is then derived from the Green's function by means of a so-called superposition formula, the latter again being characteristic of the class. For the class of heat transfer problems arising in the Gordon-type of measurement of the absorptance and emittance of a given type of electroluminescent package it is the appropriate superposition formula that leads to the integral equation formulation for  $a(T)$  and  $\epsilon(T)$ , and it is the main purpose of the present note to point out its structure. However, for purposes of reference, and also to establish definitions and notation for subsequent reports, the presentation of the appropriate superposition formulas for application to the several possible types of electroluminescent packages is preceded by a brief elementary review (without, however, any details of rigor) of the Green-type of analysis<sup>3,4</sup> as it applies to the equation of heat conduction the discussion being streamlined, as is presently customary, by use of the Dirac  $\delta$ -function technique. Although the Gordon experiment involves a one-dimensional geometry, in interests of perspective, the three-dimensional situation will be discussed first.

## 2. Three Dimensional Case, Homogeneous Medium

Consider, therefore, a homogeneous, isotropic medium occupying a region  $V$  bounded by a surface  $S$ . The medium is to have (in cgs units) a thermal conductivity  $k$ , a specific heat  $c$ , and a density  $\rho$ , all assumed temperature independent.

Throughout  $V$  one will imagine to exist a continuous distribution of heat sources of strength  $\eta(\vec{r}, t)$  ergs per  $\text{cm}^3$  per second. Let the temperature (Kelvin) as a function of position and time be specified by  $T(\vec{r}, t)$ . On  $S$  one of the following types of conditions will be assumed:

(A) Prescribed surface temperature:

$$T = \phi(u, v, t) \quad (1)$$

on  $S$ , where  $u$  and  $v$  are curvilinear coordinates in  $S$ ,

(B) Prescribed heat flux across  $S$ :

$$-\hat{n} \cdot \nabla T = \psi(u, v, t) \quad (2)$$

(C) The "radiation" condition (corresponding to Newton's law of cooling):

$$k \hat{n} \cdot \nabla T = -h\{T - T_0(u, v, t)\} \quad (3)$$

where  $h$  is the so-called "other conductivity" of the material, and  $T_0(u, v, t)$  is the temperature of the environment just adjacent to  $S$ ,

(D) The mixed condition:  $S$  may be divided into several parts on each of which one of the previous three conditions prevails.

Finally, one supposes known the temperature distribution throughout  $V$  at the instant  $t = 0$ . That is,

$$T(\vec{r}, 0) = \chi(\vec{r}) \quad (\text{say}) \quad (4)$$

Then, under initial condition (4), and for any one of the above boundary conditions, one inquires as to the subsequent temperature distribution  $T(\vec{r}, t)$  ( $t > 0$ ).

From the references one knows that in this homogeneous, isotropic case the flow of heat is governed by the equation

$$K \nabla^2 T - \frac{\partial T}{\partial t} = - \frac{\eta}{c_p} \quad (5)$$

where

$$K = \frac{k}{c_p}$$



is the thermal diffusivity of the medium. That is, the problem being posed is that of solving the non-homogeneous, linear partial differential Equation (5) under any of the above four boundary conditions and with initial condition (4). The functions  $\phi, \psi$  and  $\chi$  being arbitrary, each of the boundary conditions defines an infinite class of problems, and it is for just such a circumstance that the Green-type of analysis permits great economy of effort. The Green's function for each class is defined as the solution of (5) for a singular source function, and for the corresponding homogeneous boundary and initial conditions. More specifically, for problems of class (A) one seeks a function  $G(\vec{r}, \vec{r}', t-t')$ , points  $\vec{r}$  and  $\vec{r}'$  both lying in  $V$ , meeting the following requirements:

$$KV^2 G - \frac{\partial G}{\partial t} = - \delta(\vec{r}-\vec{r}')\delta(t-t') \quad (6)$$

$$G = 0 (\vec{r}, \vec{r}' \text{ in } V; t \leq t') \quad (7)$$

$$G = 0 (\vec{r} \text{ on } S; \text{ all } t) \quad (8)$$

That is,  $G(\vec{r}, \vec{r}', t-t')$  is the "response" of the medium when initially "quiescent", its boundary being always maintained "quiescent", to the introduction of a concentrated parcel of  $cp$  ergs of heat at the point  $\vec{r}'$ , at the instant  $t'$ . For problems of classes (B) and (C) boundary condition (8) is to be replaced respectively by

$$\hat{n} \cdot \nabla G = 0 (\vec{r} \text{ on } S; \text{ all } t) \quad (9)$$

and

$$k \hat{n} \cdot \nabla G = -h G (\vec{r} \text{ on } S; \text{ all } t). \quad (10)$$

For problems of class (D) the pertinent condition (8), (9) or (10) must be applied to each of the several sections into which  $S$  is divided. The existence of the Green's function and, in particular, its implied dependence upon  $t$  and  $t'$  through their difference, should of course be rigorously proved, but will be here accepted on intuitive grounds.

Now, considering only class (A) problems for the moment, let it be supposed that for a given material, of given size and shape, the Green's function has been determined. Then for preassigned functions  $\phi(\mu, \nu, t)$  and  $\chi(\vec{r})$  the solution of the corresponding class (A) problem is obtained as follows. First let it be noted that the so-called adjoint Green's function  $\tilde{G}(\vec{r}, \vec{r}', t-t')$ , defined by the equation

$$\tilde{G}(\vec{r}, \vec{r}', t-t') \equiv G(\vec{r}, \vec{r}', t'-t),$$

satisfies the differential equation

$$KV^2\tilde{G} + \frac{\partial\tilde{G}}{\partial t} = -\delta(\vec{r}, \vec{r}')\delta(t-t'), \quad (11)$$

the initial condition

$$\tilde{G} = 0, \quad (\vec{r}, \vec{r}' \text{ in } V; t > t') \quad (12)$$

and the boundary condition

$$\tilde{G} = 0, \quad (\vec{r} \text{ on } S; \text{ all } t). \quad (13)$$

Now, substituting from Equations (5) and (11) into Green's second identity; namely

$$\int_V (\tilde{G}V^2T - TV^2\tilde{G}) dV = \int_S (\tilde{G} \frac{\partial T}{\partial n} - T \frac{\partial \tilde{G}}{\partial n}) dS, \quad (14)$$

applying the sifting property of  $\delta(\vec{r}-\vec{r}')$ , and taking note of condition (13), one obtains

$$\begin{aligned} T(\vec{r}', t)\delta(t-t') &= \int_V (\tilde{G} \frac{\partial T}{\partial t} + T \frac{\partial \tilde{G}}{\partial t}) dV \\ &+ \frac{1}{c\rho} \int_V \tilde{G}(\vec{r}, \vec{r}', t-t') \eta(\vec{r}, t) dV - K \int_S T \frac{\partial \tilde{G}}{\partial n} dS. \end{aligned}$$

Now, integrating both sides with respect to  $t$  from 0 to  $t' + \epsilon$  where  $\epsilon > 0$ , applying the sifting property of  $\delta(t-t')$ , passing to the limit as  $\epsilon \rightarrow 0$ , introducing boundary and initial conditions and, finally, reverting back to  $G$  in place of  $\tilde{G}$  one obtains the superposition formula for class (A) problems, namely

$$\begin{aligned}
 T(\vec{r}', t') &= \int_V G(\vec{r}, \vec{r}', t) \chi(\vec{r}) dV \\
 &+ \frac{1}{c\rho} \int_V dV \int_0^t G(\vec{r}, \vec{r}', t-t') \eta(\vec{r}, t) dt \\
 &- K \int_S dS' \int_0^t \phi(u', v', t') \frac{\partial G}{\partial u'} dt' \quad \dots \dots \dots (15)
 \end{aligned}$$

the volume and surface integrals now being carried out in  $r' = \text{space}$ .

A similar deviation can be given for problems of classes (B), (C) and (D). The corresponding superposition formulas differ from (13) only in the surface integral term. This term for these respective cases is found to be as follows: For class (B) problems,

$$K \int_S dS' \int_0^t G(\vec{r}, \vec{r}', t-t') \psi(u', v', t') dt';$$

for class (C) problems,

$$- K \int_S dS' \int_0^t T_0(u', v', t') \frac{\partial G}{\partial u'} dt',$$

and for class (D) problems.

$$\begin{aligned}
 &- K \int_{S_1} dS' \int_0^t \phi(u', v', t') \frac{\partial G}{\partial u'} dt' \\
 &+ K \int_{S_2} dS' \int_0^t G(\vec{r}, \vec{r}', t-t') \psi(u', v', t') dt' \\
 &- K \int_{S_3} dS' \int_0^t T_0(u', v', t') \frac{\partial G}{\partial u'} dt'
 \end{aligned}$$

where, in the last expression,  $S_1$ ,  $S_2$ , and  $S_3$  are the portions of  $S$  over which are the portions of  $S$  over which boundary conditions of types (A), (B) and (C) maintain respectively.

### 3. Three-Dimensional Case, Composite Medium

Of greater pertinence to the heat transfer problems arising in the Gordon experiment with electroluminescent packages is the case of composite media. Consider, therefore, a region  $V$ , bounded by a surface  $S$ , occupied by two homogeneous,

isotropic media separated by an interface  $S_3$  (Figure 1). Let the sets of pertinent physical parameters of the two media be  $(k_1, c_1, \rho_1, K_1)$  and  $(k_2, c_2, \rho_2, K_2)$  respectively. Also, let the subregions they occupy be  $V_1$  and  $V_2$ ,  $V_1$  being bounded by  $S_1$  and  $S_3$ , and  $V_2$  by  $S_2$  and  $S_3$ . Finally, let  $n_{12}$  be the unit normal to  $S_3$  directed from  $V_1$  to  $V_2$ .

Now the theory of parabolic differential equations shows that, for any problem in any one of the above four classes to have a unique solution in the present case of a composite medium, two additional specifications must be made concerning conditions at the interface. One of these follows from the physical requirement that the temperature must be everywhere finite, except, possibly, at locations of ideal singular sources. This leads to the condition that the heat flux be continuous across  $S_3$ . That is,

$$k_1 \left( \frac{\partial T}{\partial n_{12}} \right)_1 = k_2 \left( \frac{\partial T}{\partial n_{12}} \right)_2 \quad \dots \dots \dots (16)$$

As to a second condition, one possible choice is that the temperature itself be continuous across  $S_3$  or, briefly, that

$$T_1 = T_2 \text{ at } S_3 \quad \dots \dots \dots (17)$$

the subscripts here, as in Equation (14), being an abbreviation for the usual limiting process. In practice, however, when the scale of observation does not permit including in the mathematical analysis the binding region (the region of transition from the one pure substance to the other), a temperature discontinuity must often be assumed. In that case, in addition to condition (16), it is customary to assume that

$$-k_1 \left( \frac{\partial T}{\partial n_{12}} \right) = H_{12}(T_1 - T_2), \quad \dots \dots \dots (18)$$

where  $H_{12}$  is the interfacial thermal conductivity, characteristic of the pair of materials and the nature of the binding. Let it now be seen how the previous superposition formulas must be modified for a two-component composite medium when, in

addition to the boundary condition on  $S$  and the initial temperature distribution throughout  $V$ , account must be taken of condition (16) and either of conditions (17) or (18). Considering first problems of class (A), with the choice of condition (17) at the interface, one now seeks a function  $T(\vec{r}, t)$  such that

$$K_1 \nabla^2 T - \frac{\partial T}{\partial t} = - \frac{\eta_1}{c_1 \rho_1} (\vec{r} \text{ in } V_1)$$

$$K_2 \nabla^2 T - \frac{\partial T}{\partial t} = - \frac{\eta_2}{c_2 \rho_2} (\vec{r} \text{ in } V_2)$$

$$T(\vec{r}, 0) = \chi(\vec{r}) \quad (\vec{r} \text{ in } V)$$

$$T(\vec{r}, t) = \phi(u, v, t) \quad (\vec{r} \text{ on } S)$$

and which, in addition, satisfies conditions (16) and (17) at  $S_3$ . The Green's function appropriate to this class of problems is defined to be such that its adjoint satisfies the following set of equations and conditions; namely

$$K_1 \nabla^2 \tilde{G} + \frac{\partial \tilde{G}}{\partial t} = - \delta(\vec{r} - \vec{r}^0) \delta(t - t^0)$$

$$(\vec{r} \text{ in } V_1, \vec{r}^0 \text{ in } V)$$

$$K_2 \nabla^2 \tilde{G} + \frac{\partial \tilde{G}}{\partial t} = - \delta(\vec{r} - \vec{r}^0) \delta(t - t^0)$$

$$(\vec{r} \text{ in } V_2, \vec{r}^0 \text{ in } V)$$

$$\tilde{G} = 0 \quad (\vec{r}, \vec{r}^0 \text{ in } V, T \geq t^0)$$

$$\tilde{G} = 0 \quad (\vec{r} \text{ on } S, \text{ all } t)$$

$$K_1 \frac{\partial \tilde{G}}{\partial n_{12}} = K_2 \frac{\partial \tilde{G}}{\partial n_{12}} \quad (\vec{r} \text{ on } S_3, \text{ all } t)$$

$$\tilde{G}_1 = \tilde{G}_2 \quad (\vec{r} \text{ on } S_3, \text{ all } t)$$

Applying Green's identity (14) separately to regions  $V_1$  and  $V_2$ , introducing the appropriate information from the above two sets of equations and conditions, and following through with the same sequence of manipulations as used before for the non-composite case, one finds that upon adding the resulting equations the contributions from the surface integrals over  $S$  cancel. Then, after taking into account the symmetry of the Green's function (which, as discussed below, prevails even in the composite case), one arrives at the superposition formula appropriate to the present case; namely

$$\begin{aligned}
 T(\vec{r}, t) &= \int_{V=V_1 \cup V_2} G(\vec{r}, \vec{r}', t) \chi(\vec{r}') dV' \\
 &+ \frac{1}{c_1 \rho_1} \int_{V_1} dV' \int_0^t G(\vec{r}, \vec{r}', t-t') \eta(\vec{r}', t') dt' \\
 &+ \frac{1}{c_2 \rho_2} \int_{V_2} dV' \int_0^t G(\vec{r}, \vec{r}', t-t') \eta_2(\vec{r}', t') dt' \\
 &= K_1 \int_{S_1} dS'_1 \int_0^t \phi(u', v', t') \frac{\partial G}{\partial n'} dt' \\
 &- K_2 \int_{S_2} dS'_2 \int_0^t \phi(u', v', t') \frac{\partial G}{\partial n'} dt' \quad \dots \dots \dots (19)
 \end{aligned}$$

It is clear how this formula generalizes when there are more than two components.

Had condition (18) been imposed in place of (17) it would have been found that precisely the same formula (19) is reached. This does not mean, of course, that the temperature distribution would be the same in the two cases. It is the Green's functions that will be different, that for the second case being functionally dependent upon the parameter  $H_{12}$  characteristic of the interface.

Finally, for problems of classes (B), (C) and (D), it is clear that the appropriate superposition formulas can be obtained from formula (19) by replacing the

surface integrals in this formula by those having the forms given above as being appropriate to these respective cases.

#### 4. Symmetry of Green's Function in Composite Case.

In a subsequent report dealing with the automatic computation of the Green's function for any of the several experimental circumstances arising in the measurement of absorptance and emittance, it will be seen that the number of operations required is considerably reduced if the Green's function is symmetric. The proof of this symmetry property seems usually given only for the case of a homogeneous medium<sup>3</sup>, and so it becomes important for the present application to be convinced that the property persists even for composite media.

In brief outline the proof for the homogeneous case runs as follows. Thus consider again a homogeneous medium occupying a region  $V$  bounded by a surface  $S$ . Let  $G(\vec{r}, \vec{r}', t-t')$  be the Green's function for any one of the previously formulated classes of boundary-value problems, and let  $\tilde{G}(\vec{r}, \vec{r}', t-t')$  be its adjoint. Apply Green's identity to the pair of functions

$$G_1 \equiv G(\vec{r}, \vec{r}_1, t-t')$$

and

$$\tilde{G}_2 \equiv \tilde{G}(\vec{r}, \vec{r}_2, t-t'').$$

$\vec{r}_1$  and  $\vec{r}_2$  being arbitrary points in  $V$ , thus obtaining

$$\begin{aligned} & \int_V [G_1(KV^2\tilde{G}_2) - \tilde{G}_2(KV^2G_1)] \\ &= K \int_S (G_1 \frac{\partial \tilde{G}_2}{\partial n} - \tilde{G}_2 \frac{\partial G_1}{\partial n}) dS. \end{aligned}$$

Now, if the homogeneous correlative (which, by definition, the Green's function must satisfy) of any of the above four types of boundary conditions is introduced into the surface integral, it will be found to vanish. Hence Eq. (20) becomes

$$\int_V [\tilde{G}_1(KV^2\tilde{G}_2) - \tilde{G}_2(KV^2\tilde{G}_1)]dv = 0$$

Then, replacing  $(KV^2\tilde{G}_1)$  by  $[\frac{\partial \tilde{G}_1}{\partial t} - \delta(\vec{r}-\vec{r}')\delta(t-t')]$  and  $(KV^2\tilde{G}_2)$  by

$[-\frac{\partial \tilde{G}_2}{\partial t} - \delta(\vec{r}-\vec{r}'')\delta(t-t'')]$ , integrating over all  $t$ , and, finally, taking into account the sifting property of the  $\delta$ -functions, one obtains

$$G(\vec{r}_2, \vec{r}_1, t''-t') = \tilde{G}(\vec{r}_1, \vec{r}_2, t'-t'').$$

But by definition of the adjoint, this may be written

$$G(\vec{r}_2, \vec{r}_1, t''-t') = G(\vec{r}_1, \vec{r}_2, t''-t'), \quad \dots \dots \dots (21)$$

the desired symmetry property.

The proof will not be outlined which substantiates the validity of (21) for the composite case, provided condition (16) and either of conditions (17) or (18) are imposed at the interface. Consider again the two-component case of Figure 1, and apply the same sequence of manipulations just outlined separately to each of the subregions  $V_1$  and  $V_2$ . Two cases will have to be considered, according as points  $\vec{r}_1$  and  $\vec{r}_2$  lie in the same subregion or not. In either case, the following will be the outcome of the calculations. In the resulting equations, by virtue of the boundary conditions, the respective integrals over  $S_1$  and  $S_2$  will vanish, leaving only contributions from  $S_3$ . However, when these equations are added, the latter contributions will be found to cancel because of the required (alternative) choices of interfacial conditions. Integrating the resulting equation over all  $T$ , and taking account of the sifting property of the  $\delta$ -functions, one again obtains property (19). It is clear that the argument may be extended to the case of more than two components.



# 6. Conclusion: The One-Dimensional Composite Case.

The desired superposition formula for the one-dimensional geometry of the Gordon experiment can be generated from that of the three-dimensional case as follows. Taking the case of a two-component layer, one allows the region V of Figure 1 to approach the form of a truncated circular cylinder the ratio of whose diameter to its length becomes infinite. Let the axis of the cylinder coincide with the x-axis of a rectangular coordinate system, and let the interface coincide with the yz-plane. Further, let the source strength  $\eta$  depend spatially solely upon x. Then, in the limit, the temperature also becomes spatially dependent solely upon x and it is easily seen that for class (A) problems formula (19) reduces to

$$\begin{aligned} T(x,t) = & \int_{-a}^b G(x,x',t)T(x',0)dx' \\ & + \frac{1}{c_1\rho_1} \int_{-a}^0 dx' \int_0^t G(x,x',t-t')\eta_1(x',t')dt' \\ & + \frac{1}{c_2\rho_2} \int_0^b dx' \int_0^t G(x,x',t-t')\eta_2(x',t')dt' \\ & + K_1 \int_0^t T(-a,t') \left( \frac{\partial G}{\partial x'} \right)_{x'=-a} dt' \\ & - K \int_0^t T(b,t') \left( \frac{\partial G}{\partial x'} \right)_{x'=b} dt' \quad \dots \dots \dots (22) \end{aligned}$$

where a and b are respectively the thickness of the layers  $V_1$  and  $V_2$ . It is evident how formula (22) generalizes in case there are more than two components in the multilayer.

It is formula (22) (and the corresponding formulas for the other classes of boundary-value problems) that has been the specific target of the foregoing review and which will form the starting point for the formulation of various heat transfer problems that arise in the measurement of absorptance and emittance by the Gordon procedure.

References

- (1) G. E. Gordon, Rev. Sci. Instr., 31, 1204 (1960).
- (2) William J. Thaler, Edward J. Finn, Paul A. Treado, Jamil Nakhleh,  
Temperature Dependence of the Solar Absorptance and Thermal Emittance  
of Copper, Gold, Nickel and Silver. (To be published by the Journal  
of Applied Optics).
- (3) Phillip M. Morse and Herman Feshbach, Methods of Theoretical Physics,  
McGraw-Hill Book Company, Inc., New York (1953), Part I, Chapter VII.
- (4) H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford  
University Press (1947), Chapter XIII.

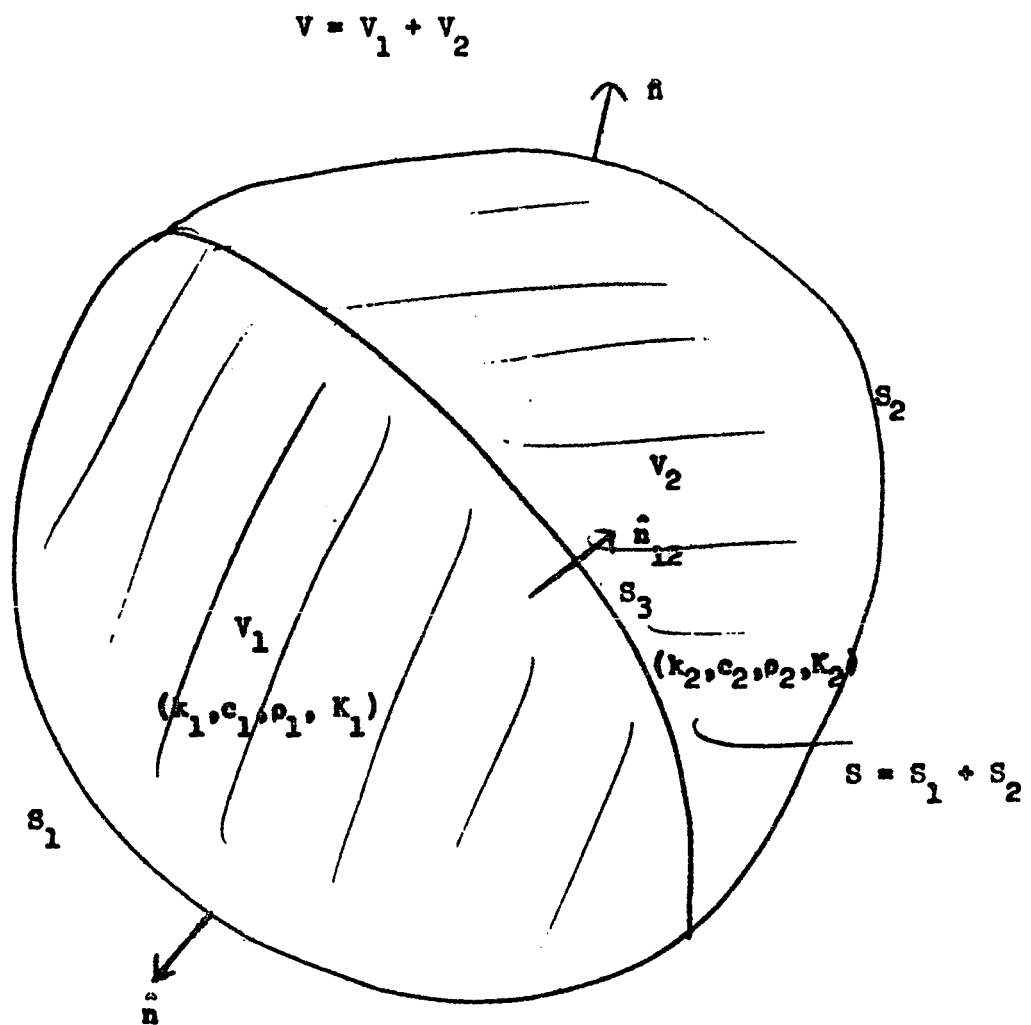


Figure 1

# Determination of Absorptance and Emittance of Electroluminescent Material from Heating and Cooling Data for an Idealized Model

## 1. Statement of Problem

Experiment has shown that significant temperature gradients exist within an electroluminescent package during the heating and cooling cycles of a measurement of its absorptance and emittance. Moreover, at least in the case of some samples, the electroluminescent material itself is not accessible for a direct measurement of its temperature. The data processing problem is then the somewhat difficult one of determining from front and back temperatures the absorptance and emittance of the material as a function of its temperature; the latter, however, not being directly measured. The present is a progress report on a study being carried out to develop methods for this determination. In order to lay bare the essential mathematical features of the problem, the following very much simplified model of an electroluminescent package, under certain hypothetical, idealized experimental circumstances, has first been studied. The model and the hypothetical experimental circumstances will later be modified and extended so as to include physically relevant factors being presently omitted.

A thin layer of solid state material (of the order of a micron or less in thickness) is enclosed between two layers of the same dielectric material of thickness  $a$  and  $b$  cms, respectively (Figure 1). Let  $k$ ,  $c$ ,  $\rho$ , and  $K = k/c\rho$  be, respectively (in cgs units) the thermal conductivity, the specific heat, the density, and the thermal diffusivity of the dielectric material, all assumed temperature independent over the temperature range of the measurement.

Just prior to an instant  $t = 0$  the sample will be assumed to have a uniform temperature  $T_0$ . At  $t = 0$  its right hand face is exposed to a beam of simulated

solar radiation, a certain portion of which (depending upon the reflection coefficient of the dielectric material) is transmitted into the layer. It will be assumed that any other source of external radiation (such as from the liquid-nitrogen-cooled shroud) may be neglected. Let the intensity of the transmitted beam be  $I$ , and let it be assumed that a negligible amount of this is absorbed by the dielectric. The film will have some transparency and reflectivity, but it will be assumed that the portion of  $I$  transmitted into layer #1 and the portion reflected back into layer #2 both pass, without experiencing any absorption or further reflection, into the outer space.

Designating the temperature (Kelvin) as a function of position and time by  $T(x,t)$ , and letting  $x = 0$  be the location of the film, the film will then absorb radiation at the rate

$$\alpha(T_f) I A'$$

where  $\alpha(T_f)$  is its absorptance, indicated as a function of the film temperature  $T_f = T(0,t)$ , and  $A'$  its projected area. Simultaneously it will emit thermal radiation at the rate

$$2 \epsilon(T_f) \sigma T_f^4 A,$$

where  $\epsilon(T_f)$  is its hemispherical emittance,  $\sigma$  the Stefan-Boltzmann constant, and  $A$  its actual area. It is assumed that none of this emitted radiation is absorbed by the dielectric. Now, neglecting the heat capacity of the film by comparison with that of the surrounding dielectric material, the net amount of heat

$$\dot{q}_f^{(1)}(t) = \alpha(T_f) I A' - 2\epsilon(T_f) \sigma T_f^4 A \quad \dots \quad (1)$$

must then be conducted away into the surrounding dielectric layers.

Finally, let it be supposed that, simultaneously with the exposure to simulated solar radiation, a voltage (DC or AC) is impressed across the package, but by means of electrodes whose radiative and thermal-conductive effects may be neglected. However, the dielectric will be allowed some lossiness at power frequencies so that

a continuous distribution of sources of strength

$$\eta(x,t) = \frac{Ri^2}{a+b} \text{ (ergs per cm per sec)} \quad . . . . . (2)$$

will be assumed to exist throughout the dielectric. Here  $R$  is the resistance of the slab to axially directed current flow and  $i$  the root-mean-square current (both in cgs units),  $R$  being assumed-temperature independent over the temperature range of the measurement.

Then, for a certain passage of time, from  $t = 0$  to  $t = \tau$  (say), the front and back temperatures  $T^{(1)}(b,t)$  and  $T^{(1)}(-a,t)$  are recorded, the superscript referring to heating cycle data.

Next, at the instant  $t = \tau$  the beam of incident radiation is suppressed, but the voltage across the sample maintained. The front and back temperatures  $T^{(2)}(b,t)$  and  $T^{(2)}(-a,t)$  are again recorded, the superscript now referring to cooling cycle data. Assuming the film resistance to be negligible, the film now performs as a sink of radiation, whose strength (reckoned as a source) is

$$\eta_f^{(2)}(t) = -2 \epsilon(T_f) T_f^4 A_c \quad . . . . . (3)$$

It is being assumed that no hysteresis effects are present - hence the same choice of notation for emittance as a function of film temperature in both Eqs. (2) and (3).

With the above model for an electroluminescent package, and under these hypothetical experimental conditions, the problem being posed for solution is the following: Determine the functions  $\alpha(T_f)$  and  $\epsilon(T_f)$  from the data  $T^{(1)}(-a,t)$ ,  $T^{(2)}(b,t)$ ,  $T^{(2)}(-a,t)$  and  $T^{(2)}(b,t)$  assuming, of course, that all pertinent parameters like thermal diffusivity, specific heat, reflection coefficient, etc., are known.

## 2. Integral Equation Formulation

Consider the following boundary-value problem in heat conduction theory. One is given a slab of homogeneous material of cross-sectional area  $A$ , of parameters  $k, c, \rho, K$ , with its end faces at positions  $x = -a$  and  $x = b$ . At the instant  $t = 0$  its temperature distribution  $T(x, 0)$  is supposed known. Also let it be supposed that a continuous distribution of heat sources of strength  $n(x, t)$  ergs per cm per sec exists throughout the material. Then in terms of  $n(x, t)$  the initial temperature distribution, and the temperature gradients  $(\partial T / \partial x)_{x=a}$  at the end faces, let the temperature distribution  $T(x, t)$  for  $t > 0$  be desired.

In a previous report it was shown, among other things, that the solution of the problem just formulated (a class (B) problem of that report) is given by the superposition formula

$$\begin{aligned} T(x, t) = & \int_{-a}^b G(x, x', t) T(x', 0) dx' \\ & + \frac{1}{cpA} \int_{-a}^b dx' \int_0^t G(x, x', t-t') n(x', t') dt' \\ & + K \int_0^t G(x, -a, t-t') \left[ \frac{\partial T(x', t')}{\partial x'} \right]_{x'=-a} dt' \\ & - K \int_0^t G(x, b, t-t') \left[ \frac{\partial T(x', t')}{\partial x'} \right]_{x'=b} dt' \end{aligned}$$

where  $G(x, x', t-t')$  is the type B Green's function for the slab of material and is assumed known. If a portion of the sources may be regarded as localized at the plane  $x = 0$ , and having a strength  $n_f(t)$  ergs per sec, then (retaining the symbol  $n(x, t)$  for the continuous portion) this last formula must clearly be modified to read

$$\begin{aligned} T(x, t) = & \int_{-a}^b G(x, x', t) T(x', 0) dx' \\ & + \frac{1}{cpA} \int_{-a}^b dx' \int_0^t G(x, x', t-t') n(x', t') dt' \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{cpA} \int_0^t G(x, 0, t-t^0) \eta_f(t^0) dt^0 \\
 & + K \int_0^t G(x, a, t-t^0) \left[ \frac{\partial T(x^0, t^0)}{\partial x^0} \right]_{x^0=a} dt^0 \\
 & - K \int_0^t G(x, b, t-t^0) \left[ \frac{\partial T(x^0, t^0)}{\partial x^0} \right]_{x^0=b} dt^0 \dots \dots \dots (4)
 \end{aligned}$$

Now let formula (4) be applied to the present idealized model of an electro-luminescent package. First let it be noticed that under the above made assumption that the dielectric material does not participate in the absorption and emission of thermal radiation, the temperature gradients at the end faces must necessarily be assumed to vanish during both the heating and cooling cycles of the measurement. The last two integrals in Eq. (4) will thus vanish, and for the heating and cooling process the formula reduces respectively to the two equations

$$\begin{aligned}
 T^{(1)}(x, t) &= T_0 \int_{-a}^b G(x, x^0, t) dx^0 \\
 &+ \frac{Ri^2}{(a+b)cpA} \int_{-a}^b dx^0 \int_0^t G(x, x^0, t-t^0) dt^0 \\
 &+ \frac{1}{cpA} \int_0^t G(x, 0, t-t^0) \eta_f^{(1)}(t^0) dt^0 \dots \dots \dots (5)
 \end{aligned}$$

and

$$\begin{aligned}
 T^{(2)}(x, t) &= \int_{-a}^b G(x, x^0, t) T^{(1)}(x^0, \tau) dx^0 \\
 &+ \frac{Ri^2}{(a+b)cpA} \int_{-a}^b dx^0 \int_0^t G(x, x^0, t-t^0) dt^0 \\
 &+ \frac{1}{cpA} \int_0^t G(x, 0, t-t^0) \eta_f^{(2)}(t^0) dt^0 \dots \dots \dots (6)
 \end{aligned}$$



where in Eq.(6) it is to be noticed that  $T^{(1)}(x, \tau)$  is the initial temperature distribution for the cooling run.

Now evaluate Eq.(5) for  $x = -a$  obtaining

$$\begin{aligned} T^{(1)}(-a, t) &= T_0 \int_{-a}^b G(-a, x', t) dx' \\ &+ \frac{R_1^2}{(a+b)c\rho A} \int_{-a}^b dx' \int_0^t G(-a, x', t-t') dt' \\ &+ \frac{1}{c\rho A} \int_0^t G(-a, 0, t-t') \eta_f^{(1)}(t') dt' \quad \dots \dots \quad (7) \end{aligned}$$

But the left-hand side of this last equation; namely, the function  $T^{(1)}(-a, t)$ , is assumed given experimental data. Hence Eq.(7) is seen to be a linear integral equation for  $\eta_f^{(1)}(t)$ .

Next, assuming this integral equation to have been solved, insert the now known function  $\eta_f^{(1)}(t)$  into Eq.(5) and evaluate the latter for  $t = \tau$ . By so doing one obtains the initial temperature distribution  $T^{(1)}(x, \tau)$  for the cooling run; namely

$$\begin{aligned} T^{(1)}(x, \tau) &= T_0 \int_{-a}^b G(x, x', \tau) dx' \\ &+ \frac{R_1^2}{(a+b)c\rho A} \int_{-a}^b dx' \int_0^\tau G(x, x', \tau-t') dt' \\ &+ \frac{1}{c\rho A} \int_0^\tau G(x, 0, \tau-t') \eta_f^{(1)}(t') dt' \quad \dots \dots \quad (8) \end{aligned}$$

Next, introducing information (8) into Eq.(6) and evaluating the latter for  $x = -a$ , one obtains a linear integral equation for  $\eta_f^{(2)}(t)$ ; namely

$$\begin{aligned}
 T_f^{(2)}(-a, t) &= \int_{-a}^b G(-a, x', t) T_f^{(1)}(x', \tau) dx' \\
 &+ \frac{Ri^2}{(a+b)cpA} \int_{-a}^b dx' \int_0^t G(-a, x', t-t') dt' \\
 &+ \frac{1}{cpA} \int_0^t G(-a, 0, t-t') \eta_f^{(2)}(t') dt' \quad \dots \dots \dots (9)
 \end{aligned}$$

Assume that (9) has been solved.

Then, finally, one needs the film temperatures  $T_f^{(1)}(t) \equiv T_f^{(1)}(0, t)$  and  $T_f^{(2)}(t) \equiv T_f^{(2)}(0, t)$  for the respective runs. For this, one inserts the now known functions  $\eta_f^{(1)}(t)$  and  $\eta_f^{(2)}(t)$  into Eqs. (5) and (6) respectively, and evaluate each of the latter for  $x = 0$ , obtaining

$$\begin{aligned}
 T_f^{(1)}(t) &\equiv T_f^{(1)}(0, t) = T_0 \int_{-a}^b G(0, x', t) dx' \\
 &+ \frac{Ri^2}{(a+b)cpA} \int_{-a}^b dx' \int_0^t G(0, x', t-t') dt' \\
 &+ \frac{1}{cpA} \int_0^t G(0, 0, t-t') \eta_f^{(1)}(t') dt' \quad \dots \dots \dots (10)
 \end{aligned}$$

and

$$\begin{aligned}
 T_f^{(2)}(t) &\equiv T_f^{(2)}(0, t) = \int_{-a}^b G(0, x', t) T_f^{(1)}(x', \tau) dx' \\
 &+ \frac{Ri^2}{(a+b)cpA} \int_{-a}^b dx' \int_0^t G(0, x', t-t') dt' \\
 &+ \frac{1}{cpA} \int_0^t G(0, 0, t-t') \eta_f^{(2)}(t') dt' \quad \dots \dots \dots (11)
 \end{aligned}$$

Then, solving Eqs. (1), (3), (10) and (11) simultaneously, one determines the functions  $\alpha(T_f)$  and  $\varepsilon(T_f)$ , and the problem has been formally solved. It is to

be observed that in carrying out this last step one first picks the  $T_f$ -value. Then he determines from Eq.(10) the corresponding  $t$ -value for insertion into Eq.(1), and from Eq.(11) the corresponding  $t$ -value for insertion into Eq.(3). If  $t_1$  and  $t_2$  are these respective  $t$ -values for a preassigned value of  $T_f$ , then the corresponding values of  $\alpha$  and  $\epsilon$  are given by the equations

$$\alpha(T_f) = \frac{\eta_f^{(1)}(t_1) - \eta_f^{(2)}(t_2)}{IA^1} \quad \dots \quad (12)$$

$$\epsilon(T_f) = - \frac{\eta_f^{(2)}(t_2)}{2\sigma T_f^4 A} \quad \dots \quad (13)$$

Lastly, it is to be noted that instead of having evaluated Eqs.(5) and (6) for  $x = -a$  to obtain integral equations for  $\eta_f^{(1)}(t)$  and  $\eta_f^{(2)}(t)$ , one could as well have evaluated these equations for  $x = b$ , obtaining an alternative pair of integral equations for these two unknowns. If the present model and the assumed experimental conditions were actually valid, the two procedures, to within experimental error, should yield the same result. In fact, the extent of the agreement (or disagreement) could be used as a measure of the validity of the assumed model and the relevancy of the physical factors omitted in the analysis.

### 3. Introduction of the Green's Function; Numerical Methods Under Study

The Green's function for the present problem is well known<sup>2</sup>, and, in the present notation, is given by the series

$$G(x, x', t-t') = \frac{1}{a+b} + \frac{2}{a+b} \sum_{n=1}^{\infty} \left\{ \exp \left[ -\frac{Kn^2\pi^2(t-t')}{(a+b)^2} \right] \right\} \cos \frac{n\pi(x+a)}{a+b} \cos \frac{n\pi(x'+a)}{a+b} \quad \dots \quad (14)$$

Substituting (14) into Eqs.(7), (8), (9), (10) and (11), these become

$$T^{(1)}(-a, t) = T_0 + \frac{Ri^2 t}{(a+b)cpA} + \frac{1}{cpA(a+b)} \int_0^t \eta_f^{(1)}(t^0) dt^0$$

$$+ \frac{2}{cpA(a+b)} \sum_{n=1}^{\infty} \cos \frac{n\pi a}{a+b} \int_0^t \eta_f^{(1)}(t^0) \exp \left[ -\frac{Kn^2 \pi^2 (t-t^0)}{(a+b)} \right] dt^0$$

(7°)

$$T^{(1)}(x, \tau) = T_0 + \frac{Ri^2 \tau}{(a+b)cpA} + \frac{1}{cpA(a+b)} \int_0^{\tau} \eta_f^{(1)}(t^0) dt^0$$

$$+ \frac{2}{cpA(a+b)} \sum_{n=1}^{\infty} \cos \frac{n\pi a}{a+b} \cos \frac{n\pi(x+a)}{a+b} \int_0^{\tau} \eta_f^{(1)}(t^0) \exp \left[ -\frac{Kn^2 \pi^2 (\tau-t^0)}{(a+b)^2} \right] dt^0$$

(8°)

$$T^2(-a, t) = \int_{-a}^b T^{(1)}(x^0, \tau) \left( \frac{1}{a+b} + \frac{2}{a+b} \sum_{n=1}^{\infty} \cos \left[ \frac{n\pi(x^0+a)}{a+b} \right] \exp \left[ -\frac{Kn^2 \pi^2 \tau}{a+b^2} \right] \right) dt^0$$

$$+ \frac{Ri^2 t}{(a+b)cpA} + \frac{1}{cpA(a+b)} \int_0^t \eta_f^{(2)}(t^0) dt^0$$

$$+ \frac{2}{(a+b)cpA} \sum_{n=1}^{\infty} \cos \left( \frac{n\pi a}{a+b} \right) \int_0^t \exp \left[ -\frac{Kn^2 \pi^2 (t-t^0)}{(a+b)^2} \right] \eta_f^{(2)}(t^0) dt^0$$

(9°)

$$T_f^{(1)}(t) = T_0 + \frac{Ri^2 T}{(a+b)cpA} + \frac{1}{(a+b)} \int_0^t \eta_f^{(1)}(t^0) dt^0$$

$$+ \frac{2}{(a+b)cpA} \sum_{n=1}^{\infty} \cos \left( \frac{n\pi a}{a+b} \right) \int_0^t \eta_f^{(1)}(t^0) \exp \left[ -\frac{Kn^2 \pi^2 (t-t^0)}{(a+b)^2} \right] dt^0$$

(10°)

$$\begin{aligned}
 T_f^{(2)}(t) = & \int_{-a}^b T_f^{(1)}(x', \tau) \left\{ \frac{1}{a+b} + \frac{2}{a+b} \sum_{n=1}^{\infty} \exp \left[ -\frac{Kn^2 \tau^2 t}{(a+b)^2} \right] \right. \\
 & \cos \left( \frac{n\pi a}{a+b} \right) \cos \left( \frac{n\pi(x'+a)}{a+b} \right) + \frac{R_1^2 t}{a+b} + \frac{1}{(a+b)cpA} \int_0^t \eta_f^{(2)}(t') dt' \\
 & \left. + \frac{2}{(a+b)cpA} \sum_{n=1}^{\infty} \cos^2 \left( \frac{n\pi a}{a+b} \right) \int_0^t \eta_f^{(2)}(t') \exp \left[ -\frac{Kn^2 \tau^2 (t-t')}{(a+b)^2} \right] dt' \right\}
 \end{aligned}
 \tag{11''}$$

These latter equations would form the basis for programming the automatic computation of  $u(T_f)$  and  $s(T_f)$  for the idealized model of the present discussion. For a more realistic model, the general features of the above mathematical pattern can be expected to persist - although, of course, with an increased complexity.

Regarding numerical analytical methods to be programmed for the solution of integral equations (7') and (9'), two standard methods for the numerical solution of integral equations are under study. The first, the polynomial method, consists in assuming for  $\eta_f^{(1)}(t)$ , say, the form

$$\eta_f^{(1)}(t) = \sum_{r=0}^N a_r t^r \tag{15}$$

for a suitable choice of degree  $N$ . Substituting (15) into (7'), and evaluating the resulting equation for  $n+1$  values  $t_i$  ( $i = 1, 2, \dots, n+1$ ) of  $t$  suitably distributed throughout the interval  $(0, \tau)$ , one obtains a set of  $n+1$  linear equations for determining the unknown coefficients  $a_r$  ( $r = 0, 1, \dots, n+1$ ). Similarly,  $\eta_f^{(2)}(t)$  may be determined.

The second method, the so-called method of algebraization, consists in replacing, by means of a suitable quadrature formula, the integral

$$\int_0^t \eta_f^{(1)}(t') G(-a, 0, t=t') dt'$$

by a sum

$$\sum_{r=0}^{N-m} Q_r^{(m)} \eta_f^{(1)}(t_r) G(-a, 0, t_{N-m} - t_r)$$

$$(m = N, N-1, \dots, 2, 1, 0)$$

Here  $t_0, t_1, \dots, t_N$  are  $N+1$  appropriately chosen points in the interval  $(0, \tau)$  and the  $Q_r^{(m)}$  are quadrature coefficients whose values depend upon the quadrature formula employed. This leads to a set of linear equations for the values of  $\eta_f^{(1)}(t)$  at these  $N+1$  points. Having solved for these  $\eta_f^{(1)}(t_r)$ , the sample points  $(t_r, \eta_f^{(1)}(t_r))$  may then be smoothed over by means of some suitable interpolation formula. Similarly for  $\eta_f^{(2)}(t)$ .

The maximum degree of refinement actually necessary in applying these well-established numerical analytic techniques to the present heat transfer problems, for the estimated amount of experimental error in the measurement, is an aspect of the problem being given careful study.

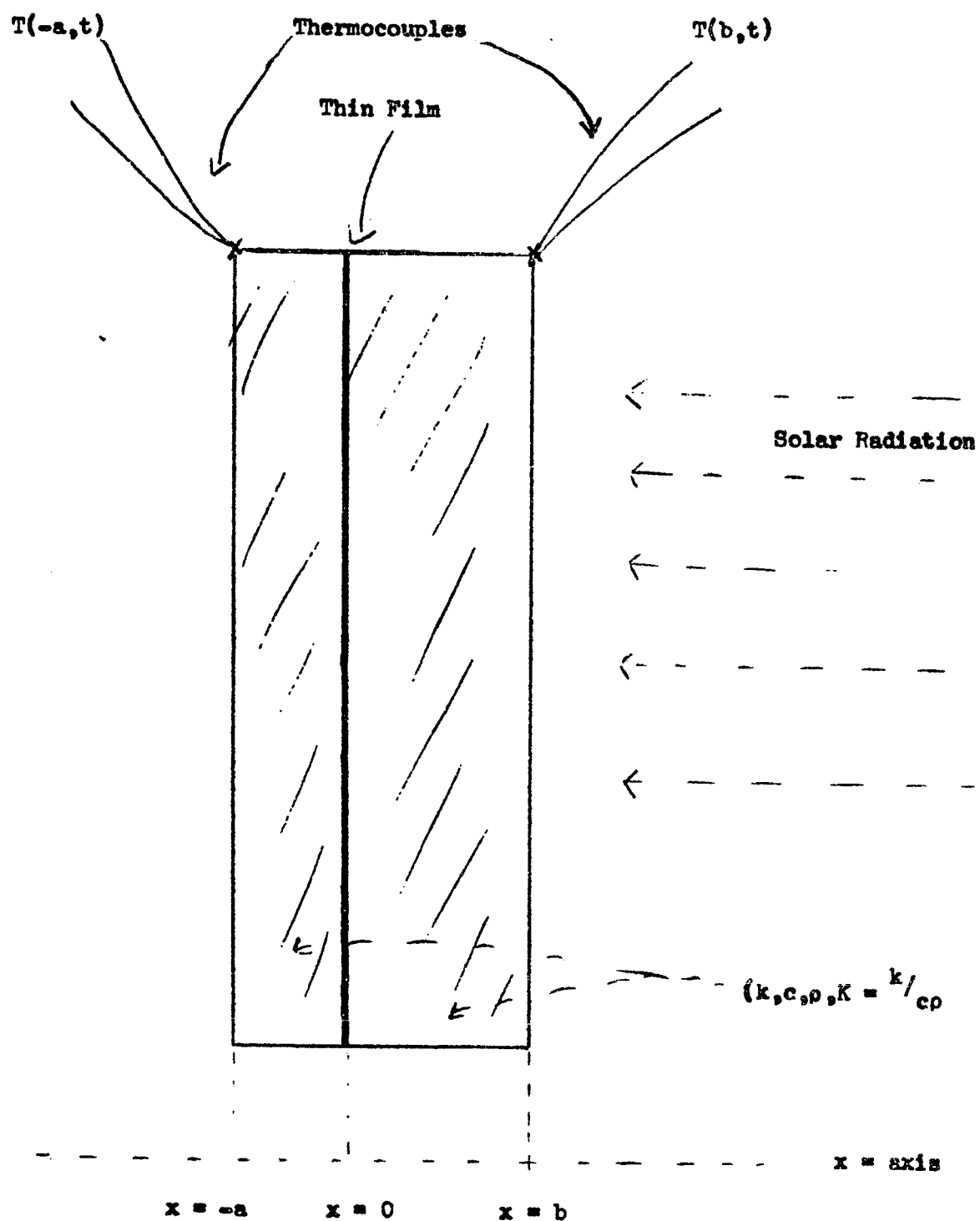


Figure 1

### References

1. "Superposition Formulas for the Temperature Gradient Problems Arising in the Measurement of Absorptance and Emittance," Summary Technical Report, May 1964, NASA Project Gs-428.
2. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford University Press (1947), Chapter XIII, Section 132.



# Absorptance and Emittance of a Thin Film of Solid State Material Enclosed Between Layers of Different Dielectric Materials

## 1. Integral Equation Formulation

In a previous report<sup>1</sup> a mathematical formalism was presented for the determination from Gordon-type data the absorptance and emittance of a thin film of electroluminescent material enclosed between two layers of the same dielectric material. This simple model was assumed in order to expose the essential mathematical structure of the problem with a minimum of interference from various physical details. With that preparation, the present report deals with the more usual case in which the film is enclosed between different dielectric materials. However, the same assumptions regarding the experimental circumstances are retained. In particular, it will still be assumed that absorption and emission by the dielectrics may be neglected, so that at the faces of the package the temperature gradients must vanish.

Thus, referring to Figure 1, a thin film of solid state material positioned at  $x = 0$  is sandwiched between two layers of dielectric material of thickness  $a$  and  $b$ . In notation of the previous report, let the sets of pertinent physical parameters of the two dielectrics be respectively  $k_1, c_1, \rho_1, K_1 = k_1/c_1\rho_1$  and  $k_2, c_2, \rho_2, K_2 = k_2/c_2\rho_2$ . Let  $G_1(x, x', t-t')$  and  $G_2(x, x', t-t')$  be the type B Green's functions<sup>2</sup> for the two layers. Then the temperature distributions  $T_1(x, t)$  and  $T_2(x, t)$  within the layers are given by the superposition formulas:

$$T_1(x, t) = \int_{-a}^0 G_1(x, x', t) T_1(x', 0) dx' \\ + \frac{1}{c_1 \rho_1 A} \int_{-a}^0 dx' \int_0^t G_1(x, x', t-t') \eta_1(x', t') dt'$$

$$\begin{aligned}
 & - K_1 \int_0^t G_1(x, -a, t-t') \left[ \frac{\partial T_1(x', t')}{\partial x'} \right]_{x'=-a} dt' \\
 & + K_1 \int_0^t G_1(x, 0, t-t') \left[ \frac{\partial T_1(x', t')}{\partial x'} \right]_{x'=0} dt' \\
 & \dots \dots \dots (1)
 \end{aligned}$$

and

$$\begin{aligned}
 T_2(x, t) &= \int_0^b G_2(x, x', t) T_2(x', 0) dx' \\
 & + \frac{1}{c_2 \rho_2 A} \int_0^b dx' \int_0^t G_2(x, x', t-t') n_2(x', t') dt' \\
 & - K_2 \int_0^t G_2(x, 0, t-t') \left[ \frac{\partial T_2(x', t')}{\partial x'} \right]_{x'=0} dt' \\
 & + K_2 \int_0^t G_2(x, b, t-t') \left[ \frac{\partial T_2(x', t')}{\partial x'} \right]_{x'=b} dt' \\
 & \dots \dots \dots (2)
 \end{aligned}$$

Applying these equations to the heating and cooling processes, taking into account the assumption of vanishing temperature gradients at  $x = -a$  and  $x = b$ , one obtains the following equations:

$$\begin{aligned}
 T^{(1)}(x, t) &= T_0 \int_{-a}^0 G_1(x, x', t) dx' \\
 & + \frac{R_1 i^2}{c_1 \rho_1 A a} \int_{-a}^0 dx' \int_0^t G_1(x, x', t-t') dt' \\
 & + K_1 \int_0^t G_1(x, 0, t-t') \left[ \frac{\partial T_1^{(1)}(x', t')}{\partial x'} \right]_{x'=0} dt' \\
 & \dots \dots \dots (3)
 \end{aligned}$$

$$\begin{aligned}
 T^{(2)}(x, t) &= \int_{-a}^0 G_1(x, x', t) T_1^{(1)}(x', \tau) dx' \\
 & + \frac{R_1 i^2}{c_1 \rho_1 A a} \int_{-a}^0 dx' \int_0^t G_1(x, x', t-t') dt'
 \end{aligned}$$

$$+ K_1 \int_0^t G_1(x, 0, t-t') \left[ \frac{\partial T_1^{(2)}(x', t')}{\partial x'} \right]_{x'=0} dt' \quad \dots \dots \dots (4)$$

$$\begin{aligned} T_2^{(1)}(x, t) &= T_0 \int_0^b G_2(x, x', t) dx' \\ &+ \frac{R_2 i^2}{c_{20}^2 Ab} \int_0^b dx' \int_0^t G_2(x, x', t-t') dt' \\ &- K_2 \int_0^t G_2(x, 0, t-t') \left[ \frac{\partial T_2^{(1)}(x', t')}{\partial x'} \right]_{x'=0} dt' \quad \dots \dots \dots (5) \end{aligned}$$

$$\begin{aligned} T_2^{(2)}(x, t) &= \int_0^b G_2(x, x', t) T_2^{(1)}(x', t) dx' \\ &+ \frac{R_2 i^2}{c_{20}^2 Ab} \int_0^b dx' \int_0^t G_2(x, x', t-t') dt' \\ &- K_2 \int_0^t G_2(x, 0, t-t') \left[ \frac{\partial T_2^{(2)}(x', t')}{\partial x'} \right]_{x'=0} dt' \quad \dots \dots \dots (6) \end{aligned}$$

On the basis of these last four equations the desired  $\alpha(T_f)$  and  $\epsilon(T_f)$  are formally determined as follows. One first notes that the  $\eta_f^{(1)}(t)$  and  $\eta_f^{(2)}(t)$  of the previous report; namely

$$\eta_f^{(1)}(t) = \alpha(T_f^{(1)}) I A' - \epsilon(T_f^{(1)}) T_f^{(1)4} A \quad \dots \dots \dots (7)$$

$$\eta_f^{(2)}(t) = - \epsilon(T_f^{(2)}) \sigma T_f^{(2)4} A \quad \dots \dots \dots (8)$$

are related to the left- and right- gradients at  $x = 0$  by the equations

$$\frac{\eta_f^{(1)}(t)}{A} = k_1 \left( \frac{\partial T_2^{(1)}}{\partial x} \right)_{x=0} - k_2 \left( \frac{\partial T_2^{(1)}}{\partial x} \right)_{x=0} \quad (9)$$

$$\frac{\eta_f^{(2)}(t)}{A} = k_1 \left( \frac{\partial T_1^{(2)}}{\partial x} \right)_{x=0} - k_2 \left( \frac{\partial T_2^{(2)}}{\partial x} \right)_{x=0} \quad (10)$$

(Here, as previously, it is being assumed that the heat capacity of the film itself is negligible.) Hence, if the four gradients occurring in Eqs.(9) and (10) can be determined,  $\eta_f^{(1)}(t)$  and  $\eta_f^{(2)}(t)$  become known. From then on the calculation for  $\alpha(T_f)$  and  $\epsilon(T_f)$  will proceed similarly to the previous case.

To determine these gradients, first introduce the heating data  $T^{(1)}(-a, t)$  into Eq.(3), obtaining

$$\begin{aligned} T_1^{(1)}(-a, t) &= T_0 \int_{-a}^0 G_1(-a, x', t) dx' \\ &+ \frac{R_1 l^2}{c \rho A a} \int_{-a}^0 dx' \int_0^t G_1(-a, x', t-t') dt' \\ &+ K_1 \int_0^t G_1(-a, 0, t-t') \left[ \frac{\partial T_1^{(1)}(x', t')}{\partial x'} \right]_{x'=0} dt' \end{aligned} \quad (11)$$

Since all quantities in this last equation are known except  $\left( \frac{\partial T_1^{(1)}}{\partial x} \right)_{x=0}$ , this equations constitutes an integral equation for that unknown. Assuming the latter to have been solved, substitute the now known  $\left( \frac{\partial T_1^{(1)}}{\partial x} \right)_{x=0}$  into Eq.(3), and evaluate the latter for  $t = 0$ . This yields the initial temperature distribution in dielectric #1 for the cooling process; namely,

$$\begin{aligned}
 T_1^{(1)}(x, \tau) = & T_0 \int_{-a}^0 G_1(x, x^0, \tau) dx^0 \\
 & + \frac{R_1 l^2}{c_1 \rho_1 A a} \int_{-a}^0 dx^0 \int_0^\tau G_1(x, x^0, \tau - t^0) dt^0 \\
 & + K_1 \int_0^\tau G_1(x, 0, \tau - t^0) \left[ \frac{T_1^{(1)}(x^0, t^0)}{\partial x^0} \right]_{x^0=0} dt^0
 \end{aligned} \tag{12}$$

Next, introducing the now known function  $T_1^{(1)}(x, \tau)$  into Eq.(4) and inserting the cooling data  $T_1^{(2)}(-a, t)$ , one obtains an integral equation for  $\left(\frac{\partial T_1^{(2)}}{\partial x}\right)_{x=0}$ ; namely

$$\begin{aligned}
 T_1^{(2)}(-a, t) = & \int_{-a}^0 G_1(-a, x^0, t) T_1^{(1)}(x, \tau) dx^0 \\
 & + \frac{R_1 l^2}{c_1 \rho_1 A a} \int_{-a}^0 dx^0 \int_0^t G_1(-a, x^0, t - t^0) dt^0 \\
 & + K_1 \int_0^t G_1(-a, 0, t - t^0) \left[ \frac{\partial T_1^{(2)}(x^0, t^0)}{\partial x^0} \right]_{x^0=0} dt^0
 \end{aligned} \tag{13}$$

It is clear that by introducing the heating and cooling data  $T_2^{(1)}(b, t)$  and  $T_2^{(2)}(b, t)$  into Eqs.(5) and (6), respectively, integral equations for  $\left(\frac{\partial T_2^{(1)}}{\partial x}\right)_{x=0}$  and  $\left(\frac{\partial T_2^{(2)}}{\partial x}\right)_{x=0}$  may similarly be found.

Finally, the film temperature as a function of time during heating and cooling may be found by evaluating Eqs.(3) and (4), respectively, for  $x = 0$ :

$$\begin{aligned}
 T_f^{(1)}(t) &= T_1^{(1)}(0, t) \\
 &= T_0 \int_{-a}^0 G_1(0, x^0, t) dx^0
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{R_1 i^2}{c_1 \rho_1 A a} \int_{-a}^0 \int_0^t G_1(0, x', t-t') dt' \\
 & + K_1 \int_0^t G_1(0, 0, t-t') \left[ \frac{\partial T_1^{(1)}(x', t')}{\partial x'} \right]_{x'=0} dx' \\
 & \dots \dots \dots (14)
 \end{aligned}$$

and

$$\begin{aligned}
 T_f^{(2)}(t) &= T_1^{(2)}(0, t) = \int_{-a}^0 G_1(0, x', t) T_1^{(1)}(x', \tau) dx' \\
 & + \frac{R_1 i^2}{c_1 \rho_1 A a} \int_{-a}^0 dx' \int_0^t G_1(0, x', t-t') dt' \\
 & + K_1 \int_0^t G_1(0, 0, t-t') \left[ \frac{\partial T_1^{(2)}(x', t')}{\partial x'} \right]_{x'=0} dt' \\
 & \dots \dots \dots (15)
 \end{aligned}$$

## 2. Introduction of the Green's Functions

The Green's functions  $G_1$  and  $G_2$  are given by the expressions

$$\begin{aligned}
 G_1(x, x', t-t') &= \frac{1}{a} + \frac{2}{a} \sum_{m=1}^{\infty} e^{-\frac{K_1 n^2 \pi^2}{a^2} (t-t')} \cos \frac{n\pi(x+a)}{a} \\
 & \cos \frac{n\pi(x'+a)}{a} \\
 & \dots \dots \dots (16)
 \end{aligned}$$

$$\begin{aligned}
 G_2(x, x', t-t') &= \frac{1}{b} + \frac{2}{b} \sum_{m=1}^{\infty} e^{-\frac{K_2 n^2 \pi^2}{b^2} (t-t')} \\
 & \cos \frac{n\pi x}{b} \cos \frac{n\pi x'}{b} \\
 & \dots \dots \dots (17)
 \end{aligned}$$

To indicate the nature of the calculation to be programmed for the automatic computation of  $u(T_f)$  and  $\epsilon(T_f)$  consider the integral equation, Eq.(12), for

$\left(\frac{\partial T_1}{\partial x}\right)_{x=0}$ . Introducing the above expression for  $G_1$ , this equation becomes

$$T_1^{(1)}(-a, t) = T_0 + \frac{R_1 i^2 t}{c_1 \rho_1 A a} + \frac{K_1}{a} \int_0^t \phi(t') dt' + \frac{2K_1}{a} \int_0^t \phi(t') \left[ \sum_{m=1}^{\infty} (-1)^m e^{-\frac{K_1 n^2 \pi^2}{a^2} (t-t')} \right] dt' \quad (18)$$

where  $\phi(t)$  stands for  $\left(\frac{\partial T_1}{\partial x}\right)_{x=0}$ .

Suppose the programming of the solution of (18) is to be based upon the polynomial method of the previous report. Then one assumes for  $\phi(t)$  the form

$$\phi(t) = \sum_{r=0}^N p_r t^r \quad (19)$$

for some suitable  $N$ , and seeks to determine the coefficients  $p_r$ . Introducing expression (19) into Eq.(18), one obtains the equation

$$\sum_{r=0}^N p_r C_r(t) = T_1^{(1)}(-a, t) - T_0 - \frac{R_1 i^2 t}{c_1 \rho_1 A a} \quad (20)$$

where

$$C_r(t) = + \frac{K_1}{a} \frac{t^{r+1}}{r+1} + \frac{2K_1}{a} \sum_{m=1}^{\infty} (-1)^m e^{-\frac{K_1 n^2 \pi^2}{a^2} t} \int_0^t t'^r e^{-\frac{K_1 n^2 \pi^2}{a^2} (t-t')} dt' \quad (21)$$

in which the integrals may be evaluated in closed form by a recursion procedure.

Next, inserting the recorded values of  $T_1^{(1)}(-a, t)$  at  $N+1$  values  $t_1, t_2, \dots, t_{N+1}$  of  $t$  suitably distributed throughout the duration  $\tau$  of the

heating run, one obtains the following set of  $N + 1$  linear equations in the  $N + 1$  unknowns  $p_0, p_1, \dots, p_N$ ; namely

$$\sum_{r=0}^N p_r C_r(t_j) = T_1^{(1)}(-a, t_j) - T_0 = \frac{R_1 l^2 t_1}{c_1 \rho_1 A a}$$

$$(j = 1, 2, \dots, N + 1) \quad \dots \dots \dots (22)$$

The integral equations for the other three gradients may be similarly reduced.

It is clear that preliminary to the programming of a calculation such as one based upon Eqs. (20), (21) and (22) one must give careful study to such matters as the appropriate choice of  $N$  for the given data  $T_1^{(1)}(-a, t)$ , the rate of convergence of the series in (21) for a dielectric layer of given thicknesses and given thermal diffusivity, etc.



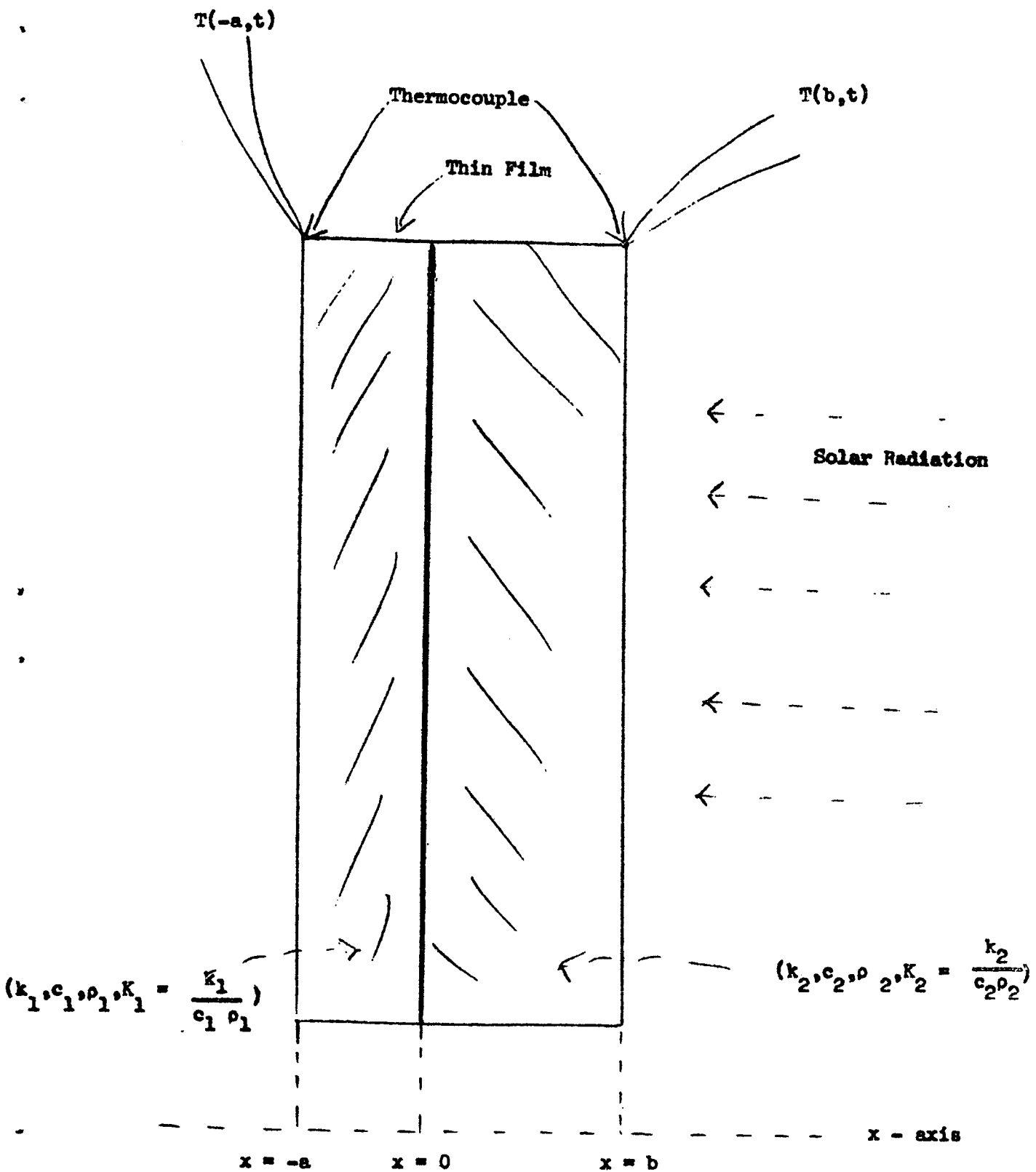


Figure 1

### References

1. "Determination of Absorptance and Emittance of Electroluminescent Materials from Heating and Cooling Data for an Idealized Model," Summary Technical Report, May 1964, NASA Project Gs-428.
2. "Superposition Formulas for the Temperature Gradient Problems Arising in the Measurement of Absorptance and Emittance," Summary Technical Report, May 1964, NASA Project Gs-428.
3. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford University Press (1947), Chapter XIII, Section 132.